Modulo Interdisciplinare CLIL
Titolo del Modulo: "Meccanica relativistica e quantistica"

Realizzato dalla prof.ssa Rita Pancari
con la collaborazione della prof.ssa Barbara Belfiore

Obiettivi:

1) Conoscere le tappe evolutive delle teorie fisiche che hanno consentito il superamento di problemi
2) Saper applicare tale strumento all'osservazione di fenomeni in particolari sistemi di riferimento nel macro cosmo e nel micro cosmo
3) Migliorare le competenze di: " essere in grado di interpretare e/o elaborare dati; contestualizzare avvenimenti scientifici, operare con algoritmi in una lingua straniera "
4) Migliorare le competenze della lingua inglese
5) Espandere il patrimonio lessicale in L1 e L2

Obiettivi Trasversali

1) Saper collaborare con i compagni e partecipare con interesse alle lezioni
2) Saper attivare strategie di apprendimento e apprendere da prospettive diverse
3) Saper riflettere su quanto svolto e individuare punti di forza e criticità
4) Saper attivare i processi cognitivi superiori: distinguere, confrontare, descrivere, sintetizzare, operare collegamenti.

Prerequisiti

Disciplinari	Linguistici
- Conoscenze di Meccanica classica, termodinamica, chimica, elettromagnetismo onde - Comprendere una situazione fisica considerando ipotesi esplicative attraverso modelli o analogie o leggi - Saper formalizzare matematicamente un problema fisico e applicare gli strumenti matematici e disciplinari specifici	- Conoscere le principali strutture linguistiche di livello intermedio. - Capire parole o espressioni scritte e orali inerenti al modulo - Enunciare in forma scritta e orale definizioni e proprietà - Eseguire correttamente istruzioni richieste

Competenze disciplinari

Conoscenze

- Problematiche non risolte classicamente
- Strutture matematiche di supporto alla fisica
- Meccanica relativistica
- Meccanica quantistica

Abilità

- Saper comprendere ed utilizzare il micro linguaggio in inglese
- Saper interpretare processi dimostrativi e risolutivi
- Saper elaborare in modo autonomo percorsi inerenti al modulo
- Saper interpretare testi di problemi e compilare questionari in inglese

Competenze linguistiche

- The first aim for the students is to be able to understand the linguistic function giving directions, understanding tasks which is used to introduce all the activities they have to carry out.

The expression used are concerned with:

- The imperative (complete, work out, find, explain, prove…)
- To have to…
- The second purpose is to know and to be able to use the microlanguage used in their activities
- The third objective is to be able to produce the language used to express the rules necessary to work out the activities.

Metodologia

- Lezione dialogata
- Problem Solving
- Lezioni multimediali di ascolto e visione
- Cooperative Learning

L'attività è stata svolta con metodologia CLIL. Tale metodologia prevede un apprendimento fondamentalmente attivo, interazionale e cooperativo. Le fasi in cui è stato diviso il modulo sono le seguenti:

Strumenti (testi, materiali, attività, risorse)

- Computer;
- Power point;
- Schede di verifica

Contenuti

MODULO 1

Unità 1 Introduzione alla relatività:

- Richiami su relatività galileiana
- Crisi della fisica classica
- Strumenti matematici
- Sistema spazio-temporale

Unità 2 Relatività speciale

- Principio di relatività ristretta
- Spazio e tempo relativi
- Composizione Velocità
- Massa
- Vettore momento-energia
- Verifiche sperimentali

Unità 3 Relatività generale

- Principio di equivalenza
- Principio di relatività generale
- Gravità e curvatura spazio-temporale
- Verifiche sperimentali

MODULO 2

Unità 1 Premesse alla teoria dei quanti:

- Continuo e discreto
- Problemi della fisica irrisolti
- Riepilogo modelli atomici

Unità 2 Meccanica quantistica:

- Dualismo onda-corpuscolo
- Principio di indeterminazione
- Equazione di Schrödinger

Fase 1 **INTRODUCTION**

a) attività motivazionale di warming up.
b) attività di verifica dei prerequisiti disciplinari mediante Brainstorming e test d'ingresso
c) attività di contestualizzazione disciplinare

Fase 2 **READING AND LISTENING**

In questa fase gli studenti hanno lavorato in piccoli gruppi secondo uno svolgimento cooperativo e socializzante, continue e mirate sono state le sollecitazioni ad interventi degli alunni per verificare la comprensione di quanto spiegato e abituarli all'uso del micro linguaggio specifico, gli è stato anche richiesto di leggere in lingua inglese parti della presentazione che ponevano questioni storico –scientifiche problematiche in modo da abituarli alla *comprehension questions.*

Fase 3 **PRACTICE**

Sono state proposte attività di consolidamento, rinforzo, approfondimento e verifica in cui gli alunni hanno adoperato le conoscenze e le abilità disciplinari e linguistiche obiettivo del modulo. Le attività proposte sono state:

a) attività di comprensione del testo;
b) attività di consolidamento/ampliamento lessicale.
c) esercizi applicativi correlati alla dimensione disciplinare e linguistica del testo.

In particolare la verifica si è svolta in itinere mediante semplici *Learning Check* con immediate soluzioni, presenti all'interno delle presentazioni e alla fine di ogni modulo si è poi effettuata una verifica sommativa che mirava a valutare il conseguimento degli obiettivi prefissati, in particolare lo sviluppo delle *thinking skills.*

Fase 4 **PRODUCTION (OUTPUT)**

Gli alunni hanno prodotto lavori presentati in lingua inglese completi di esempi sperimentali e glossari, effettuando attività di ricerca, approfondimento e di produzione linguistica.
Nella presentazione il gruppo ha scelto autonomamente come dividersi i compiti ma all'atto della presentazione hanno interagito tra loro e con la docente rispondendo a domande in lingua.

Le tematiche affrontate sono state contestualizzate nel periodo storico e hanno consentito spunti di *interdisciplinarietà,* oltre che con l'inglese anche con la matematica, essendo state trattate le geometrie non euclidee per la descrizione dello spazio tempo e delle geodetiche.

Vengono inserite di seguito schede relative ai test di ingresso e alle verifiche finali.

ENTRY TEST
"EINSTEIN'S RELATIVITY"

Answer the following questions after having read them quickly. Before you answer those questions that seem more clear, then returning on the other by fixing time limits. You have 30 minutes; each correct answer worth 1 point, each incorrect or no given answer worth zero points.

1. **Classical relativity does NOT apply to which of the following?**
   - objects moving at the speed of light
   - an object moving away from a stationary observer
   - two objects moving relative to each other
   - events in the inertial frames

2. **The inertial mass is**
   - property of attracting the masses
   - property of opposing to change its state
   - property to stand still
   - both first and second answer

3. **Which of the following best describes an inertial frame of reference?**
   - zero velocity
   - the effect of gravity on mass
   - constant velocity
   - constant acceleration

4. **In classical physics, the time depends on the reference system?**
   - no
   - yes
   - sometimes
   - only in the inertial frames

5. **According to Isaac Newton, time is:**
   - relative
   - absolute
   - dependent on which clock you use
   - flows at different paces

6. **Which physical quantity is invariant in the Galilean relativity?**
   - velocity
   - displacement
   - acceleration
   - none of the above

7. **What is the law of velocity composition according to Galilean Theory?**
   - $v' = v - v_r$
   - $v' = -v$
   - $v' = v$
   - $x't = v' + v_r$

8. **Applying a force "F" to a body of mass 'm' we obtain an acceleration 'a'. If the acceleration becomes double:**
   - F becomes 2F
   - m becomes m/2
   - "a" cannot increase
   - you do not have enough data

9. **Light is a wave**
   - mechanical
   - electromagnetic
   - of probability
   - none of the above

10. **If current in one coil becomes steady, then magnetic field lines becomes**
   - zero
   - constant
   - less than before
   - more than before

11. **Direction of induced e.m.f. in a circuit is in accordance with law of**
   - conservation of mass
   - conservation of charge
   - conservation of energy
   - conservation of momentum

12. **Magnitude of induced EMF depends on**
   - speed of motion of coil and magnet
   - number of turns of coil
   - current passing through coil
   - both first and second answer

13. **Direction of induced current always**
   - opposes the cause
   - remains same as that of cause
   - equal to cause that produces it
   - directs the cause

14. **Value of induced EMF is directly proportional to**
   - current through coil
   - voltage applied
   - no. of field lines
   - resistivity

15. **Which of the following laws is one of Maxwell's equations?**
   - Biot-Savart
   - Gauss Theorem
   - Auto induction
   - Laplace

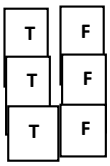
01) Lorentz transformation equations hold for (1,5p)? [] non-relativistic velocities []relativistic velocities [] all velocities []photons only	**02) An observer moving with a velocity v sees the light speed c (1,5p):** [] c-v [] c+v [] c [] v/c
03)Relativity doesn't apply to which of the following(1,5p)? []objects moving at the light speed []object moving away []object moving slowly [] objects in relative motion	**04) How time slows down at high speed? (1,5p)?** []light speed constancy [] light travels at shorter distance []light speed is relative []light travels at greater distance
05) Concerning the rest mass (1,5p): []it is associated to a stationary body []it describes energy [] it is associated to a photon []there is no-mass in light	**06) The relativistic factor γ is equal to (1,5p):** []$1/\sqrt{1-v^2/c^2}$ []$1/\sqrt{1-c^2}$ []v^2/c^2 []v/c
07) General Relativity describes (1,5p)? [] forces [] curvature [] gravitation [] geodesics	**08) The invariance of Energy – Momentum is (1,5p)?** []$p^2c^2 = m^2c^4$ []$E = mc^2$ []$E^2 - p^2c^2 = 0$ []$E^2 - p^2c^2 = m^2c^4$
09) What is the four-dimensional frame in the universe called? (1,5p): [] space-time [] universal curvature [] gravitation []nothing of the above	**10) GPS works according to (1,5p):** [] special relativity [] red shift correption [] gravity [] length contraction

a) Join the words to the meanings (9p):

Momentum-energy	In GR the world line of a particle free from all external, non-gravitational force, is a particular type of it
Rest-mass	It is an invariant quantity which is the same for all observers in all reference frames
Geodesic	It is the generalization of a classical three-dimensional vector to four-dimensional space-time

b) TRUE or FALSE (9p):

- Einstein thought that the Lorentz Transformation equations describe properties of space and time
- Two events that are simultaneous in one reference frame are necessarily simultaneous in another reference
- Massive bodies don't warp and curve their local space-time



c) Complete the sentence with the words given below (7p):

Albert Michelson built an extremely precise device called an interferometer to ……. the minute phase difference between two light ……. traveling in mutually …… directions. After several …… and refinements with assistance from Edward Morley obtained again a …… result. There is no such thing as the ……. ; the speed of light is the same for all …… observers regardless of the state of motion of the source.

1) null, 2) measure, 3) repeats, 4) waves, 5) Ether, 6) orthogonal, 7) inertial.

d) Answer the following questions (30p):

1. What is the *Minkowski chronotope*?

……

……

2. State and explain the principle of special relativity

……

……

3. What is the principle of equivalence in Relativity?

……

……

e) Solve three of the following exercises (30p)

1. A light beam travels at a speed of $3 \cdot 10^8$m/s. If the speed of sound is 340.29 m/s, what is the value of γ for sound?

2. A man embarks on the spaceship from Planet of the Apes, which travels at a speed of 0.9999837c and spends 5 years exploring space and 5 years coming back home. How much time has elapsed on Earth when he returns?

3. An electron has a rest mass of 511 keV/c^2 and is accelerated to a speed of 0.8c. What amount of mass does it gain?

4. An atomic clock is placed in a jet airplane. The clock measures a time interval of 3600 s when the jet moves with speed 0,40c. How much larger a time interval does an identical clock held by an observer at rest on the ground measure?

5. A muon is created 6km above the earth, moving towards it at 0.998c. Seen from a frame moving with the muon, what is its distance to the earth.

ENTRY TEST
"QUANTUM THEORY"

Answer the following questions after having read them quickly. Before you answer those questions that seem more clear, then returning on the other by fixing time limits. You have 30 minutes; each correct answer worth 1 point, each incorrect or no given answer worth zero points.

1. **Which subatomic particle has the smallest mass?**
   - neutron
   - proton
   - electron
   - both first and second answer

2. **What is the atomic number of an atom equal to?**
   - the number of protons in the nucleus
   - the number of neutrons in the nucleus
   - the number of electrons in the nucleus
   - the number of protons and neutrons in the nucleus

3. **What are isotopes?**
   - compounds with the same molecular formulae but different structural formulae
   - atoms with the same mass number but different atomic number
   - atoms with the same number of protons but a different number of neutrons
   - atoms with the same number of neutrons but a different number of protons

4. **Ernest Rutherford's work showed that:**
   - substances exist as atoms
   - atoms contain electrons
   - atoms have a nucleus
   - nucleus contains protons

5. **According to Thompson model:**
   - electrons rotate around the nucleus
   - electrons were in the nucleus
   - positive charge was in the centre of the atom and electron at the border
   - positive charge occupied the atom containing negative charges

6. **Which type of electromagnetic radiation travels faster?**
   - radio waves
   - gamma
   - ultraviolet
   - all of the above

7. **What can split white light into its component colours?**
   - prism
   - disperser
   - protractor
   - separation surface

8. What type of waves are light and the other forms of electromagnetic radiation?

- longitudinal
- transverse
- both first and second answer
- you do not have enough data

9. What type of electromagnetic radiation can produce photoelectric effect?

- radio waves
- gamma waves
- infrared waves
- all of the above

10. What type of radiation can associate to a blackbody spectrum?

- mechanical
- electromagnetic
- of probability
- none of the above

11. Which of these waves cannot travel through a vacuum?

- sound
- light
- probability
- electromagnetic

12. What do we measure in hertz in the waves?

- speed
- wavelength
- frequency
- period

13. What size gap causes significant diffraction?

- a gap that is sized similarly to the wavelength
- a gap that is a little larger than the wavelength
- a gap that is much larger than the wavelength
- both first and second answer

14. Every chemical element has a unique `signature' which can be revealed by:

- analysing the light, it gives off
- analysing chemical variations
- analysing resistivity to the diffusion
- analysing the incident waves

15. The light given off by atoms in a hot dilute gas forms a spectrum of:

- all colours
- only some colours
- white colour
- none of the above

01) The energy of a photon depends on its (1.5p): []speed []pressure [] amplitude [] frequency	**02) The wavelength of a particle is called (1.5p)?** [] de Broglie [] quantum [] size [] Plank
03) The photoelectric effect was explained by (1.5p): []wave nature light []particle nature light []wave nature electron [] particle nature electron	**04) Diffraction effects cannot be measured for (1.,5p):** [] light waves [] water waves []electrons []every sphere
05)In diffraction the constructive interference □ is (1.5p): [] $\propto$ senα [] $\propto$ 1/2d [] = 2dcosα [] = n/2dsenα	**06)What is called density of probability (1.5p)?** []$\Delta p/\Delta V\Delta t$ [] Ψ []$\Psi^2/\Delta t^2$ []$\Delta V\Delta t/\Delta p$
07) What did Max Planck propose to solve the black body radiator problem (1.5p)? [] radiation consist in waves [] light changes its speed []light comes in packets of energy []objects do not radiate	**08) Which of the following colors is associated with the highest temperature of a black body radiator (1.5p)?** []red []blue []green []violet
09) How does the energy of a photon change if the wavelength is doubled (1.5p)? [] is cut to one-half [] quadruples []doubles []is cut to one-fourth	**10) Rutherford's Gold Foil experiment caused a modification of (1.5p):** [] de Broglie hypothesis [] plum-pudding model [] planetary atom model [] quantum theory

a) Join the words to the meanings (9p):

Electron	A wave that can produce emission of electrical charges
Superposition	The first elementary quantum of electricity, that can produce diffraction
Photon	A quantum entity can exist in multiple possibilities of realities

b) TRUE or FALSE (9p):

- The time between the incidence and emission of a photoelectron is less than 10^9 second
- According to the Rayleigh-Jeans law the frequency diverged as it neared the ultraviolet region
- Corpuscular theories of light cannot explain diffraction and interference

T	F
T	F
T	F

c) Complete the sentence with the words given below (7p):

The Schrödinger equation is used to find the allowed……. levels of quantum ……systems (such as atoms). The associated ……Ψ gives the……. of finding the particle at a certain……. It is the time dependent equation used for describing progressive……., applicable to the motion of free…… and the time independent form of this equation used for describing standing waves, within the limits of negligible relativistic effects.

1) mechanical 2) wavefunction 3) energy 4) waves 5) probability 6) particles 7) position

d) Answer the following questions (10 points each):

4. Describe wave-particle duality:

……

……

5. State and explain the Heisenberg's Uncertainty Principle:

……

……

6. What is the quantum tunneling effect?

……

……

e) Choose three of the following questions and perform the indicated calculations (10 points each).

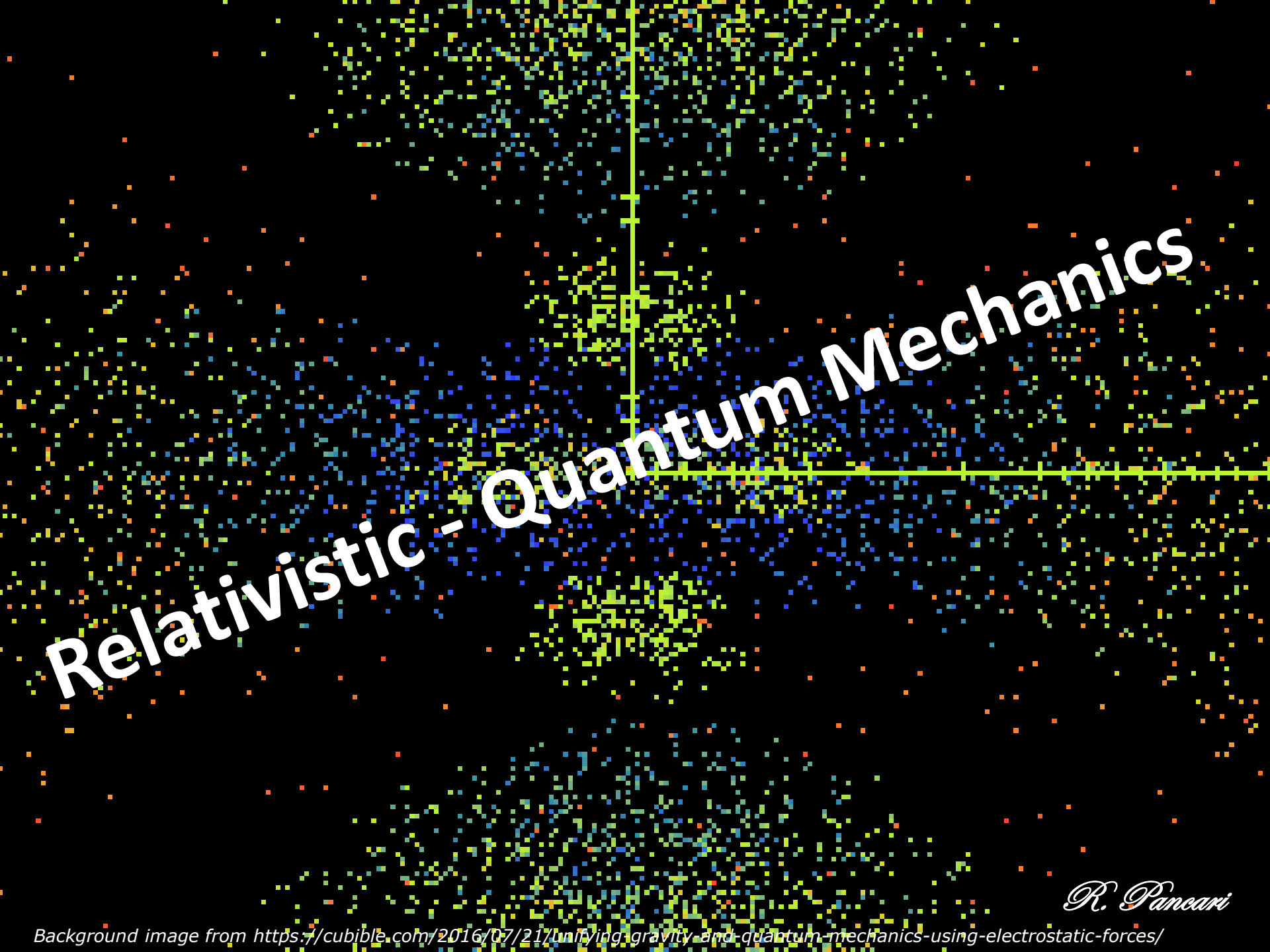
6. A harmonic oscillator has an electron in it. It is found that when the electron transitions from a state to the another, a photon is emitted with energy E = 0.356 eV. What is the angular velocity ω for this oscillator?

7. An ultraviolet radiation with a wavelength of 250 nm hits a silver sheet ($W = 6.9 \times 10^{-19}$ J). Calculate the maximum energy emitted by the electron.

8. Monochromatic X-ray, wavelengths $\lambda = 0.166$ nm, affect one KCl crystal. If the distance between the planes is 0.314nm at what angle to the surface of the crystal should be directed to the beam to be able to observe a maximum of the second order?

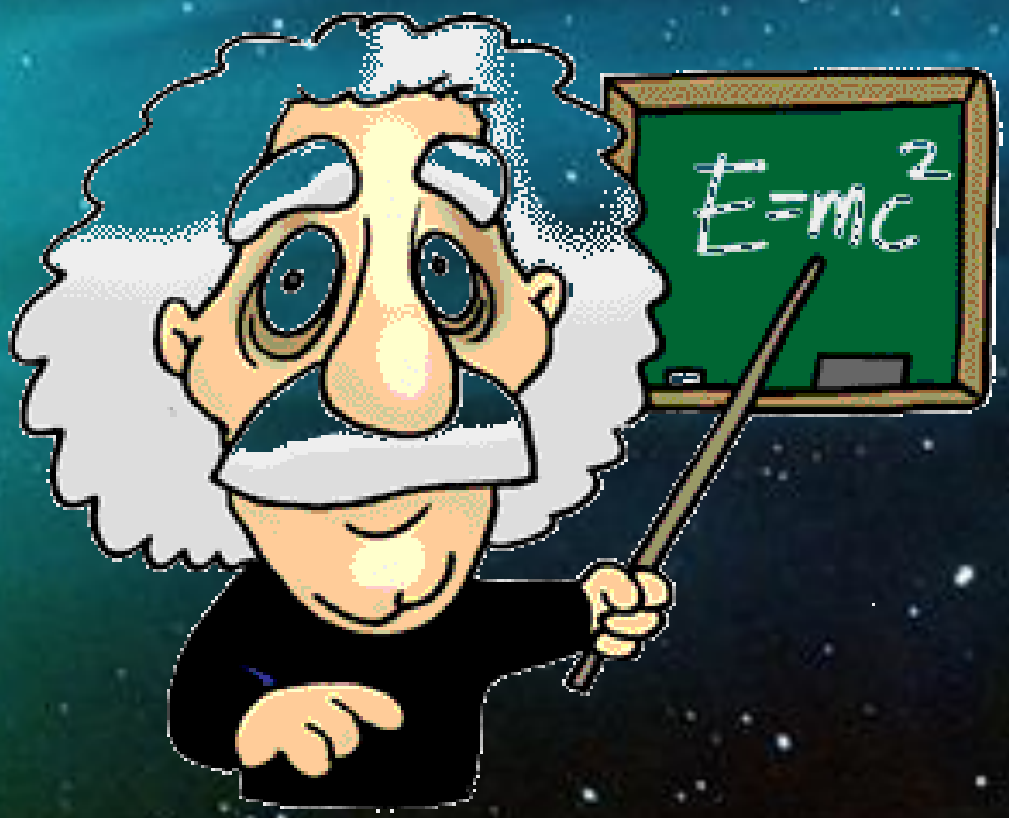
9. Compares the uncertainty about the speed of an electron ($m_e = 9.11 \cdot 10^{-31}$kg) with the uncertainty on the velocity of a water droplet ($\delta = 10^3$kg/m^3; d = 0.1 μm), confined within one size region $\Delta x = 1\mu m$.

10. A company that produces green colour laser, with a wavelength of 532 nm, declares that his 700mW power model can turn on a cigar! How many photons emit every second this pattern?



Background image from https://cubible.com/2016/07/21/unifying-gravity-and-quantum-mechanics-using-electrostatic-forces/

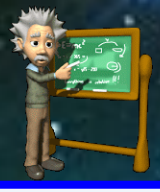
Einstein Relativity



Background image from Adam Block/Mount Lemmon SkyCenter/University of Arizona

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Einstein Relativity



- Introduction
  - Galilean Relativity
  - Classical physics crisis
    - Maxwell's equations
    - Ether
  - Relativistic math
    - Chronotope
    - Lorentz transformations



- Special Relativity
  - Kinematics
    - Velocity
    - Time and space
  - Dynamics
    - Mass
    - Energy



- General Relativity
  - Equivalence principle
  - Gravitation and curvature

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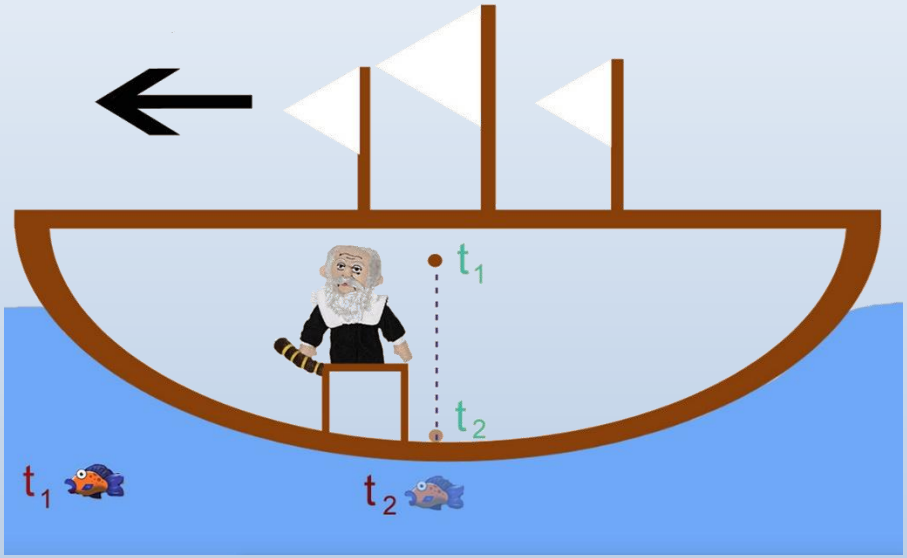
Galilean's Relativity

The principle of Galilean's relativity in 1632 states that:

the laws of the physics (classical mechanics only) are invariants into inertial reference systems

Another formulation could be expressed as follows:

- There exists an ***absolute space***, a Galilean inertial frame is a reference frame in relative uniform motion to absolute space
- All inertial frames share a ***universal absolute time***
- In the absence of external interactions, free bodies are following straight lines at constant velocity (or are at rest). Any influence that causes a free body to deviate from this state is called a force.



Galileo's experiment:
ship is moving at a constant speed to the left, the fish remains stationary relative to the Earth. Galileo drops a ball at time t_1 that hits the ground at time t_2. The fish's position is displayed at both times.

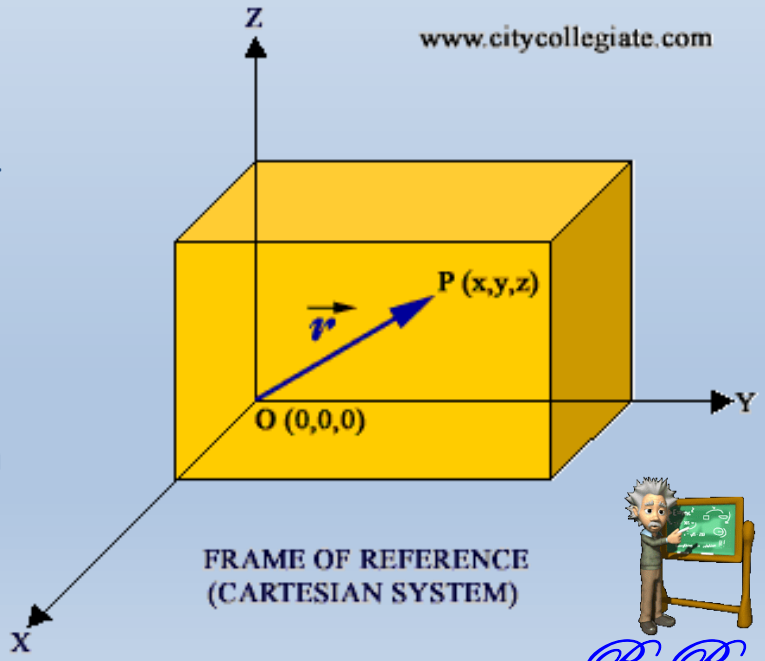


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Frames of reference

Physical observers are considered to be surrounded by a **reference frame** which is *a set of coordinate axes in terms of which position or movement may be specified* or with reference *to which physical laws may be mathematically stated*. In these frames an **event** *is something that happens independently of the reference frame that might be used to describe it*. Turning on a light or the collision of two objects would constitute an event. Frames of reference can be divided into two groups: **inertial** and **non-inertial**.

Inertial frame is a frame of reference that describes time and space homogeneously, isotopically and in a time-independent manner. *Physical laws take the same form in all inertial frames*. All inertial frames are in a state of constant, rectilinear motion with respect to one another. Measurements in one inertial frame can be converted to measurements in another by a **simple transformation** (the *Galilean transformation* in Newtonian physics and the *Lorentz transformation* in special relativity).

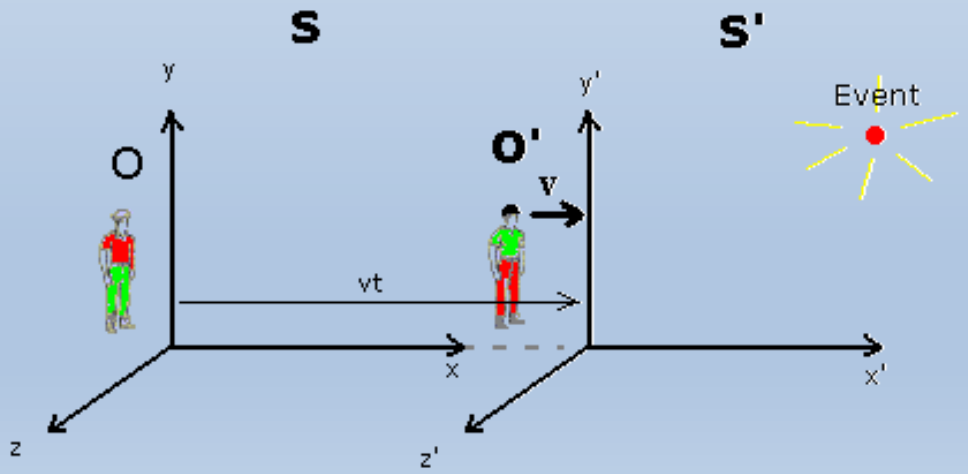


Galilean coordinate transformations

The transformations connect the space and time coordinates of an event as measured by an observer in each frame. Suppose there is a small event, such as a light being turned on, that is at coordinates in one reference frame. What coordinates would another observer, in another reference frame moving relative to the first at velocity along the axis assign to the event? The coordinates refer to the positions and timings of the event that are measured by each observer and, for simplicity, the observers are arranged so that they are coincident at t=0.

According to Galilean Relativity the transformations are:

Transformation of Coordinates



The observers are moving at a relative velocity of v and each observer has their own set of coordinates (x,y,z,t) and (x',y',z',t'). What coordinates do they assign to the event?

$$\begin{cases} x' = x - vt \\ y' = y \\ z' = z \\ t' = t \end{cases}$$



R. Pancari

Galilean's velocity transformations

The use of Newtonian physics to calculate velocities and other physical variables has led to Galilean Relativity being called Newtonian Relativity in the case where conclusions are drawn beyond simple changes in coordinates. Both observers will therefore agree on the mass being the same and so F = ma is equivalent in both frames. In his postulate Principia Mathematica, Newton defined 'absolute motion' as "the translation of a bod from one absolute place to another..." But what does he mean by "absolute place"? He also defined 'absolute time' as "absolute true and mathematical time, of itself, and from its own nature flows equally without regard to anything external".... Which just means that time is independent of space .

The velocity transformations for the velocities in the three directions in space are, according to Galilean relativity:

$$\begin{cases} u'_x = u_x - v \\ u'_y = u_y \\ u'_z = u_z \end{cases}$$

This result is known as the **classical velocity addition theorem** and summarises the transformation of velocities between two Galilean frames of reference.



$\vec{u'}_x = \vec{u}_x - \vec{v}$

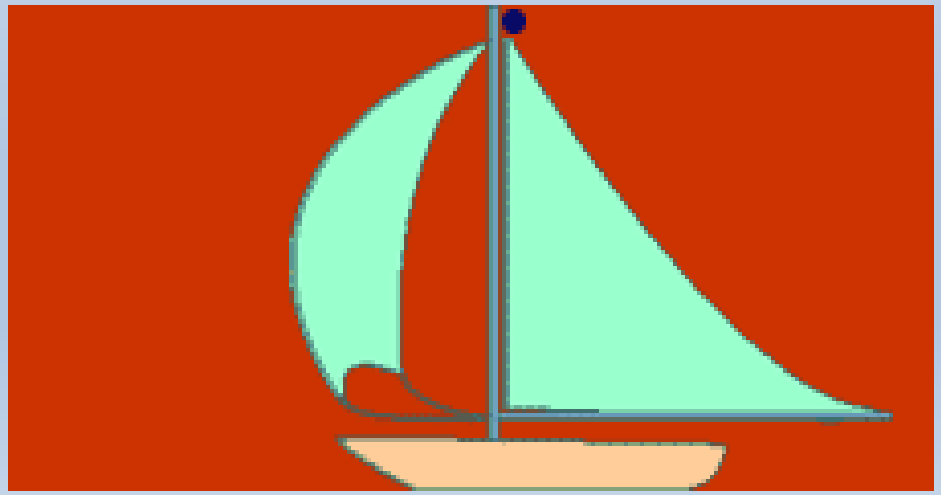


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Galilean's Relativity Limits

Inertial reference systems can be in quiet or in uniform rectilinear motion, consequently there aren't absolute reference systems; for example the world is moving around the sun of an accelerated motion and the sun is moving around the galaxy of an accelerated motion and so on. And there isn't a universal absolute time, as the successes of modern physics shows us.

Although aware that motion is relative, Newton believed that there was some fixed frame of reference to which all other motions could be compared "the Ether" and Maxwell described electromagnetism with laws that did not satisfy the Galilean relativity.





R. Pancari

Learning Check 1

If a boat is moving at constant speed relative to water, on a trip between two cities, the ride upstream lasts t_1= 6h and the ride downstream lasts t_2= 3h. What time (t') will the boat need to finish the ride downstream with its engines turned off?





R. Pancari

Solution 1

Galilean Transformations

Given

$t_1 = 6$ h

$t_2 = 3$ h

Variables

$v_1 = v_{boat} - v_{river}$

$v_2 = v_{boat} + v_{river}$

$v_3 = v_{river}$

Requests

$t' = ??$

Work

x = constant →

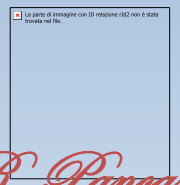
$v_1 t_1 = v_2 t_2 = v_3 t_3$

So taking the three possible combinations we obtain 3 equations of 3 variables:

t_3, v_{boat}, v_{river}

Solving them we obtain the correct result=

$$t' = \frac{2t_1 t_2}{t_1 - t_2} = 12 \text{ h}$$



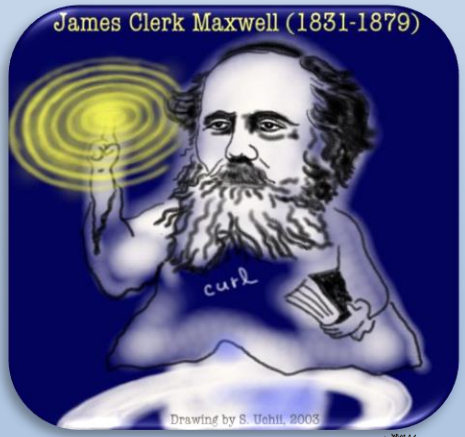
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Classical physics crisis

Towards the end of the nineteenth century it seemed that the conceptual edifice of physics was completed. Newtonian mechanics on the one hand and Maxwell's theory of electromagnetism, on the other hand, seemed to provide the key to the interpretation and prediction of all phenomena, from the motion of the planets to the behavior of electric charges, but……

"Between the late 19th century and early 20th century some experimental observations challenge the classical conception of the physical world: on one hand the behavior of light respect to the different reference frames moving relative to one another, on the other the first insights regarding the particle structure of the energy released or absorbed in the form of radiation by various bodies. It is in the 20th century that these queries, and many others that derived from them, find their answers, in either the Theory of Relativity or in Quantum Mechanics...".[1]





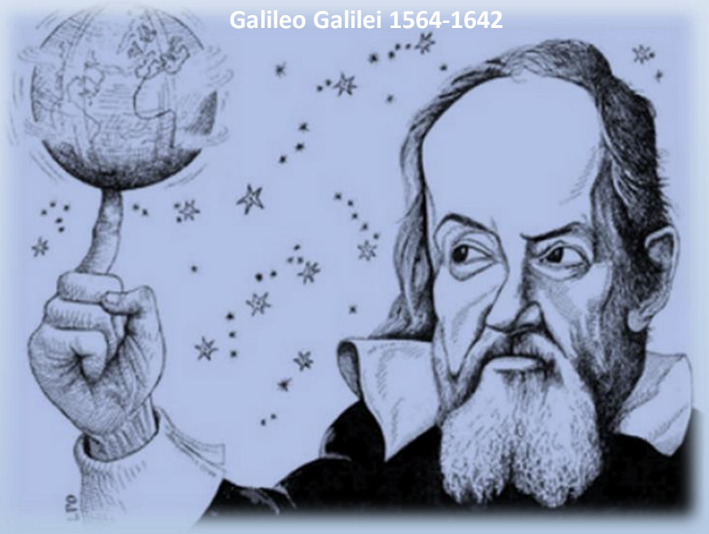


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[1] These words were spoken by the Italian physicist Edoardo Amaldi in 1955 to summarize the extraordinary events that revolutionized scientific thought in the 20th century and that marked the transition from classical to modern physics.

Unjustified phenomena

- *The principle of Galilean relativity seemed to be at odds with the Maxwell equations that change their shape when switching from one inertial reference system to another.*
- *The independence of the speed of light from the chosen reference system, demonstrated experimentally by Michelson and Morley in 1881, put into crisis the Galilean principle of velocity composition.*


Galileo Galilei 1564-1642

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Maxwell's Equations

In the nineteenth century, Maxwell had admirably summarized the entire electromagnetism in his famous differential equations (linear) partial differential written in vector form in which it appeared as a universal constant velocity of light in vacuum; these equations represent the Gauss law on electrical flow, the absence of magnetic monopoles, the induction law or Faraday-Neumann-Lenz and Ampere's law.

The sets of four equations of electrodynamics thus modified are known as Maxwell's equations:

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\varepsilon_o} \quad \text{Gauss's law (electric)}$$

$$\oint_S \mathbf{B} \cdot d\mathbf{A} = 0 \quad \text{Gauss's law in magnetism}$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt} \quad \text{Faraday's law}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_o \mathrm{I} + \varepsilon_o \mu_o \frac{d\Phi_E}{dt} \quad \text{Ampere-Maxwell law}$$

These four equations together with the Lorentz force law encapsulate classical electrodynamics.



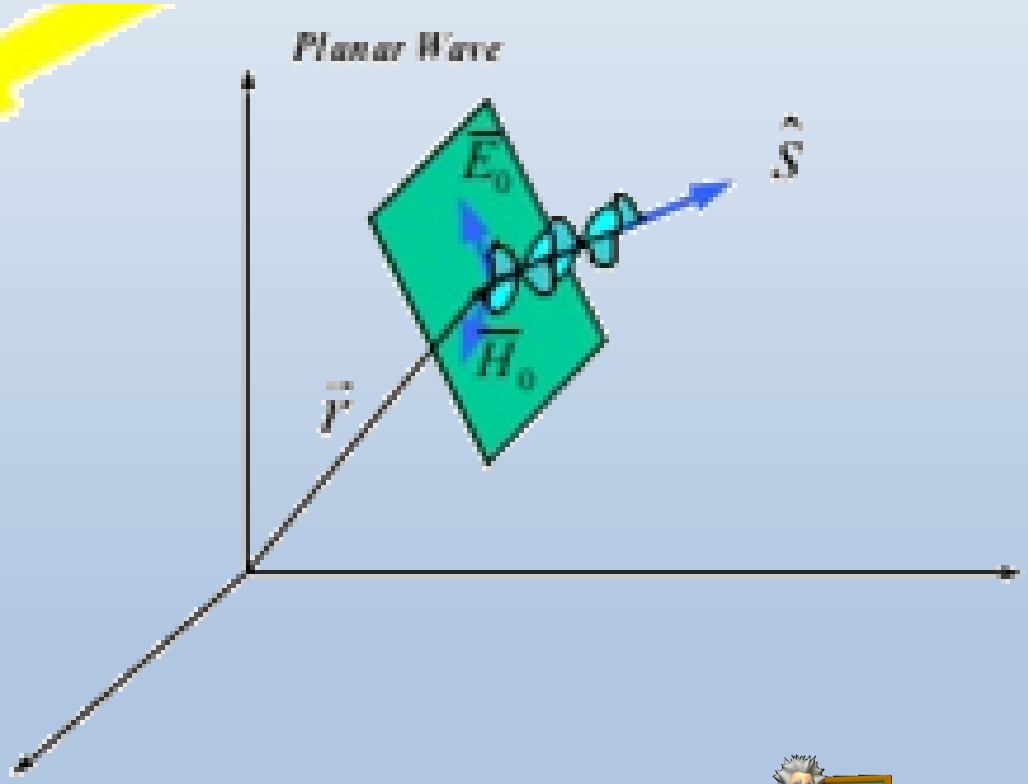
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Maxwell's Equations

Maxwell presupposed the existence of an absolute space in which were worth his equations.

So either:

- the transformations of Galileo were "wrong" (in this case, all the mechanics so-called "classical" or "Newtonian" had to be re-founded);
- or Maxwell's equations were wrong for electromagnetism;
- or the principle of Galilean relativity was not worth (while for mechanics it was).





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Speed of Light

In the Maxwell equations describing electric and magnetic fields and their propagation, three constants are normally used. One is the speed of light c, and the other two are the electric permittivity of free space ε_0 and the magnetic permeability of free space, μ_0. The magnetic permeability of free space is taken to have the exact value $\mu_0 = 4\pi \times 10^{-7} N / A^2$



With the magnetic permeability established, the electric permittivity takes the value given by the relationship $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

where the speed of light c is given by

$$c = 2.99792458 \times 10^8 m/s(exact) \approx 3 \times 10^8 m/s$$

This gives a value of free space permittivity

$$\varepsilon_0 = 8.854187817 \times 10^{-12} F/m \approx 8.85 \times 10^{-12} F/m$$

which in practice is often used in the form

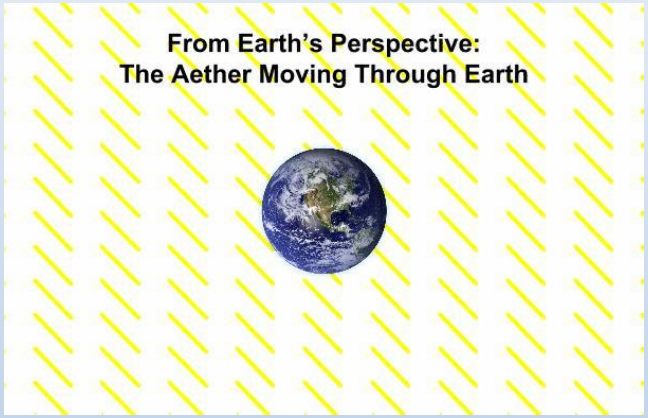
$$k = \frac{1}{4\pi\varepsilon_0} = 8.987552 \times 10^9 Nm^2/C^2$$ = Coulomb's constant

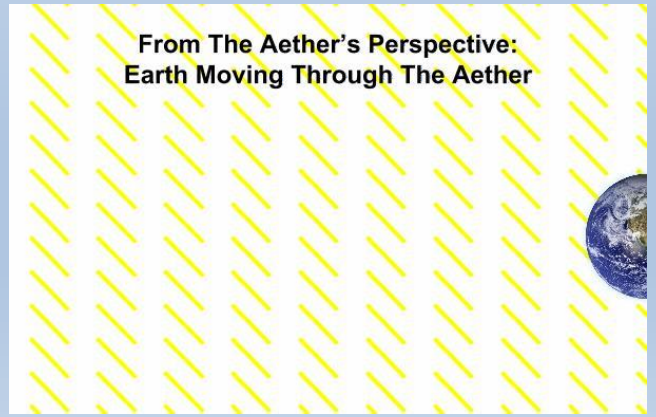


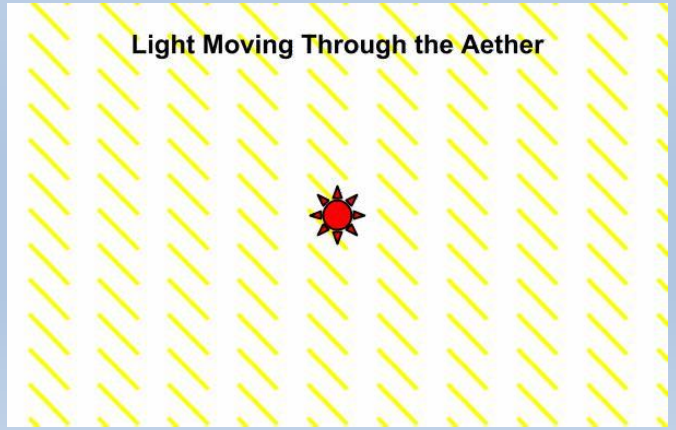
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Maxwell's Equations and Ether

If the Maxwell equations are valid and the simple classical addition of velocities applies then there should be a preferred reference frame, the frame of the stationary ether. The preferred reference frame would be considered the true zero point to which all velocity measurements could be referred.









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Learning Check 2

A. What do describe Maxwell's Equations?

1) dynamics 2) electromagnetism

B. Which of Maxwell's Equations describes the field created by an electric current?

1) Gauss's Law 2) Ampere's Law

C. Which of Maxwell's Equations describes the flow of a vector?

1) Gauss's Law 2) Ampere's Law



R. Pancari

Solution 2

A. What do describe Maxwell's Equations?

2) electromagnetism

B. Which of Maxwell's Equations describes the field created by an electric current?

2) Ampere's Law

C. Which of Maxwell's Equations describes the flow of a vector?

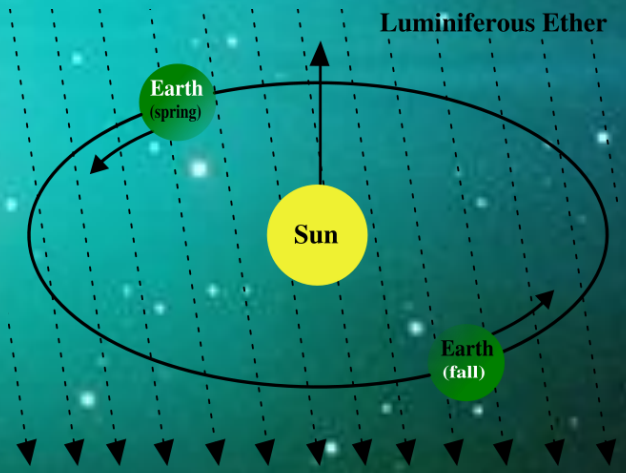
1) Gauss's Law



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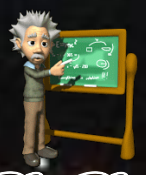
The Ether

In the late 19th century, *luminiferous aether, aether or ether*, meaning light-bearing aether, was the postulated medium for the propagation of light. It was invoked to explain the ability of the apparently - based light to propagate through empty space, something that waves should not be able to do.



Ether had to have such a low density that the planets could move through it without loss of energy. It also had to have an elasticity to support the high velocity of light waves. Ether was proposed as an absolute reference system in which the speed of light was this constant and from which other measurements could be made.

- The Michelson-Morley experiment was an attempt to show the existence of ether.



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Michelson – Morley interferometer

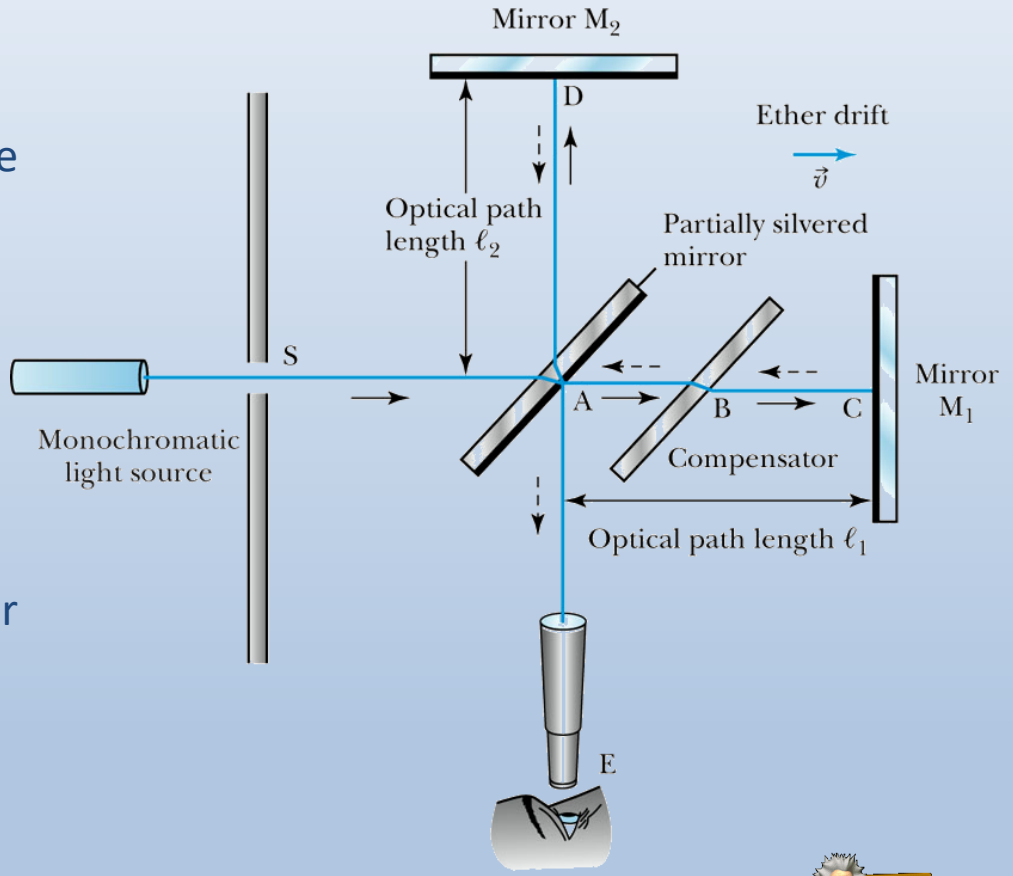
Albert Michelson (1852–1931) built an extremely precise device called an interferometer to measure the minute phase difference between two light waves traveling in mutually orthogonal directions.

DESCRIPTION

1. AC is parallel to the motion of the Earth inducing an “ether wind”

2. Light from source S is split by mirror A and travels to mirrors C and D in mutually perpendicular directions

3. After reflection the beams recombine at A slightly out of phase due to the “ether wind” as viewed by telescope E.



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Experiment Analysis

Assuming the Galilean Transformation

There is e relation between the time t_1 from A to C and back and the time t_2 from A to D and back:

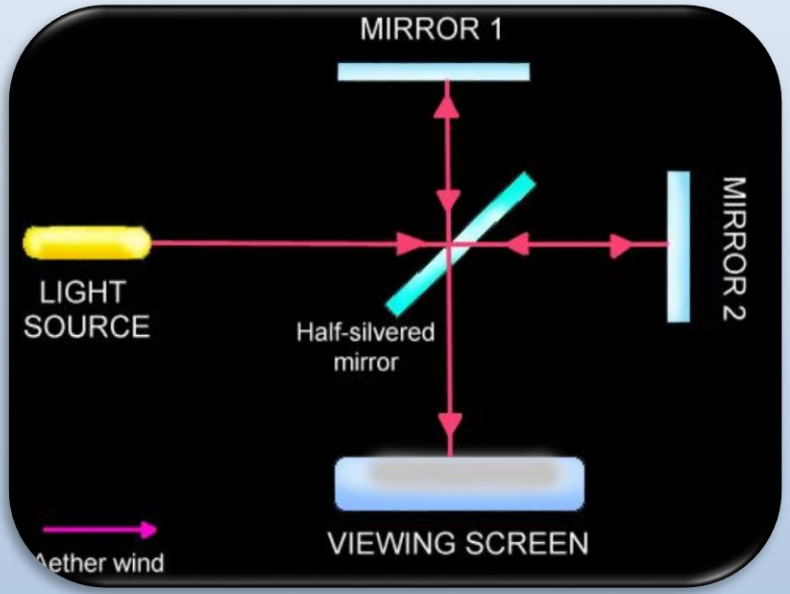
$$\Delta t = t_2 - t_1$$

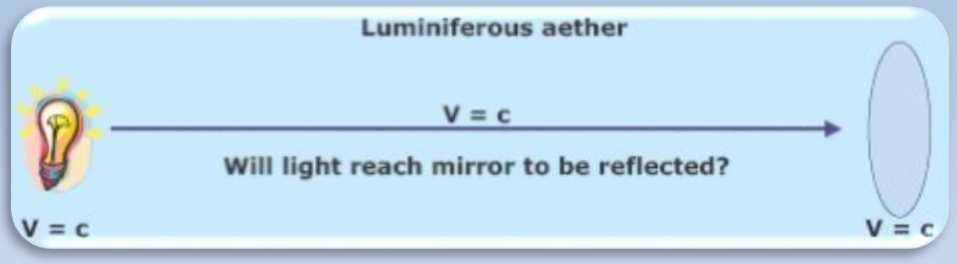
Upon rotating the apparatus, the optical path lengths ℓ_1 and ℓ_2 are interchanged producing a different change in time:

$$\Delta t' = t'_2 - t'_1$$

assuming v/c << 1, a time difference between rotations, , is given by:

$$\Delta t' - \Delta t \approx v^2(\ell_1 + \ell_2)/c^3$$







Results

- Using the Earth's orbital speed as:

$$V = 3 \times 10^4 \text{ m/s}$$

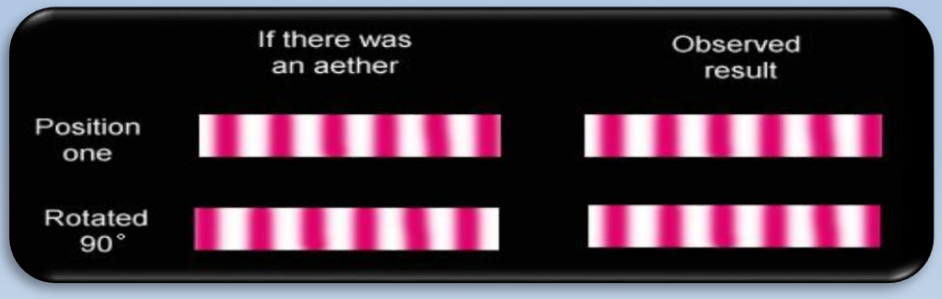
together with

$$\ell_1 \approx \ell_2 = 1.2 \text{ m}$$

So that the time difference becomes

$$\Delta t' - \Delta t \approx v^2(\ell_1 + \ell_2)/c^3 = 8 \times 10^{-17} \text{ s}$$

- Although a very small number, it was within the experimental range of measurement for light waves.



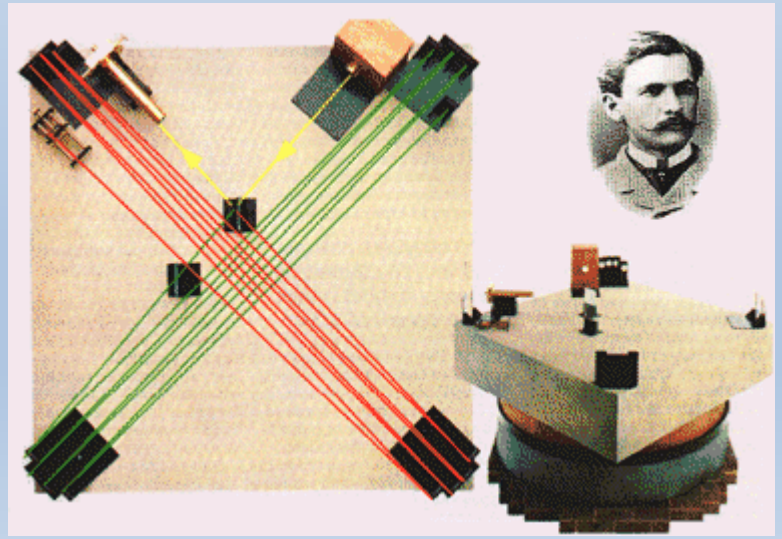


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Conclusions

After several repeats and refinements with assistance from Edward Morley obtained again a null result. These experiments are now referred to as “the kicking-off point for the theoretical aspects of the ***Second Scientific Revolution***”.

- There is no such thing as the Ether
- The speed of light is the same for all inertial observers regardless of the state of motion of the source.
- This very last observation is of particular importance, since it is incompatible with the Galilean-transformation invariance requirement. Great minds, such as Poincaré, Lorentz or Einstein tackled this problem.





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Learning Check 3

A. What does measure the Michelson-Morley's interferometer?

1) light speed 2) light interference

B. What is the result of Michelson-Morley experiment?

1) the ether existence 2) light speed invariance

C. What are the consequences derived from the "failure" of the Michelson-Morley experiment?

1) new physical theories 2) the ether nonexistence



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Solution 3

A. What does measure the Michelson-Morley's interferometer?

2) light interference

B. What is the result of Michelson-Morley experiment?

2) light speed invariance

C. What are the consequences derived from the "failure" of the Michelson-Morley experiment?

1) the ether nonexistence



R. Pancari

Geometry in the Relativity

Einstein suggested a new way of seeing reality: a phenomenon occurs in three dimensions, at a certain moment of history, that is placed in space-time. In relativity there are four coordinates: three determine the position in space and a fourth determines the position in time.

Three years after the publication of Einstein, Hermann Minkowski simplified the theory by providing a geometric interpretation that allowed to accept the strange Einstein's calculations. This geometry, non-Euclidean, used one of the most important ***Riemann's*** ideas : *the nature of a mathematical space is determined by the way you measure the distance*; so we can say that the distance formula determines the nature of the geometry.





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Riemannian geometry

In the nineteenth century Riemann revolutionized the geometry, before Einstein's theory, he spoke of **n-dimensional varieties**, consisting of ordered tuples of real numbers, for example the plan was bidimensional and the space three-dimensional. He introduced the concepts of distance and geodesic, in these varieties.
The function called ***distance*** in the space is defined as follows:

$$ds^2 = a_{11}\,dx^2 + a_{22}\,dy^2 + a_{33}\,dz^2 + 2a_{12}\,dx\,dy + 2a_{13}\,dx\,dz + 2a_{23}\,dy\,dz$$

If we consider the plan:

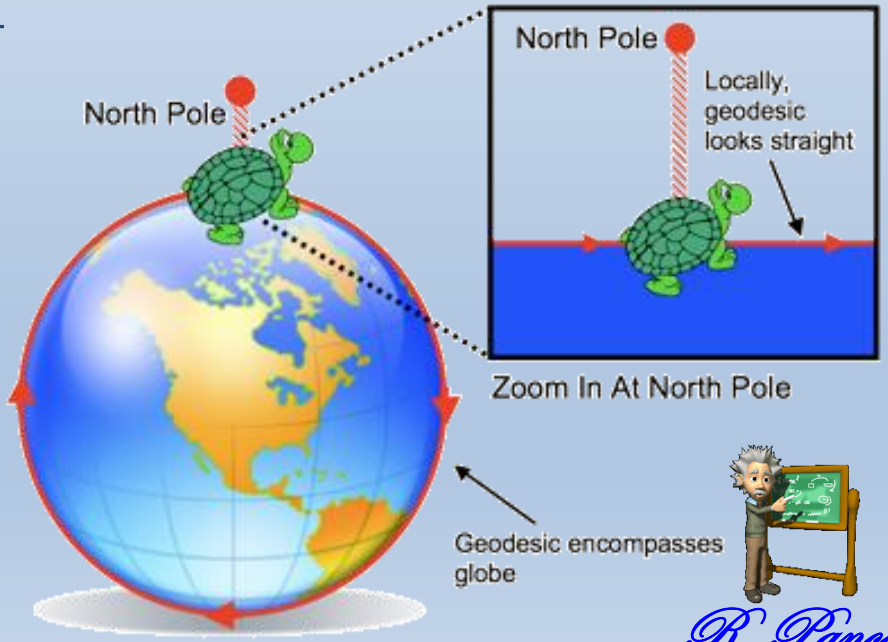
$$ds = \sqrt{a_{11}dx^2 + 2a_{12}dxdy + a_{22}dy^2}$$

were:

$a_{11}>0;\ a_{22}>0;\ a_{11}a_{22}> a_{12}$

If $a_{11} = a_{22} = 1$ and $a_{12} = 0$ we have the Euclidean plan

Geodesic is a line that represents the shortest path between two points, for example meridians on the earth are geodesic lines



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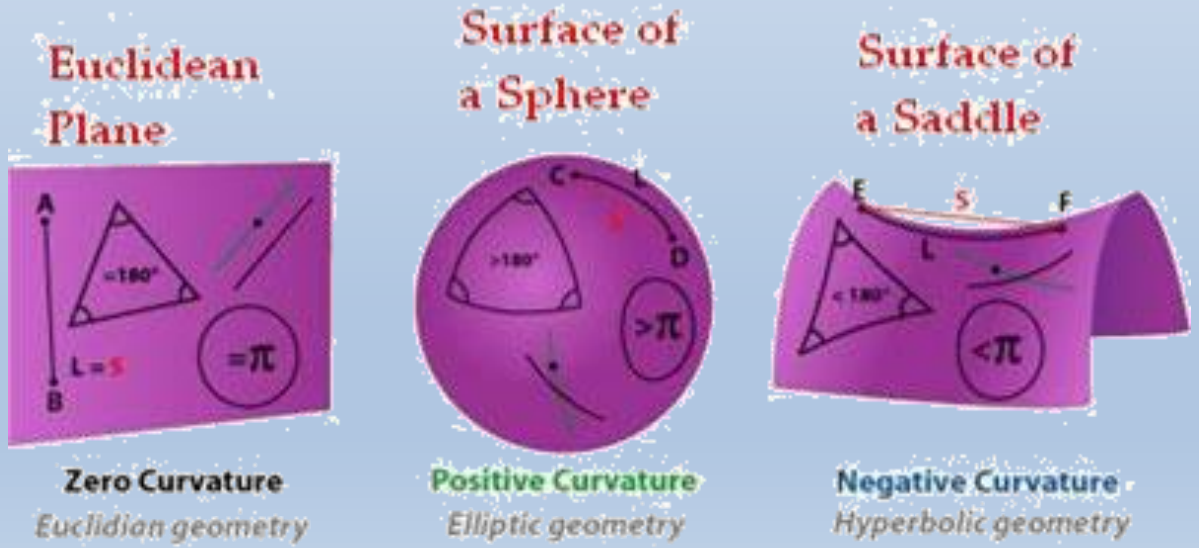
Non Euclidean Geometries

The function ***ds*** is associated with the ***curvature of the variety***, that may be:

- constant curvature → ds = constant and the space has the same properties;
- variable curvature → ds ≠ constant and the space has the variable properties.

The non Euclidean Geometries are associated with the constant curvature:

- ***ds < 0; hyperbolic geometry (Lobachevski);***
- ***ds = 0; parabolic geometry (Euclid);***
- ***ds > 0; elliptic geometry (Riemann)***





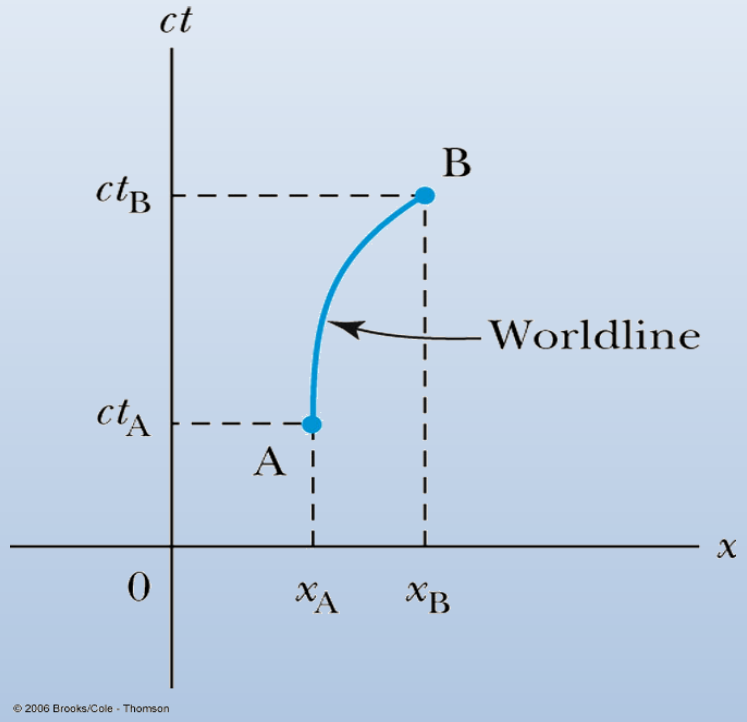
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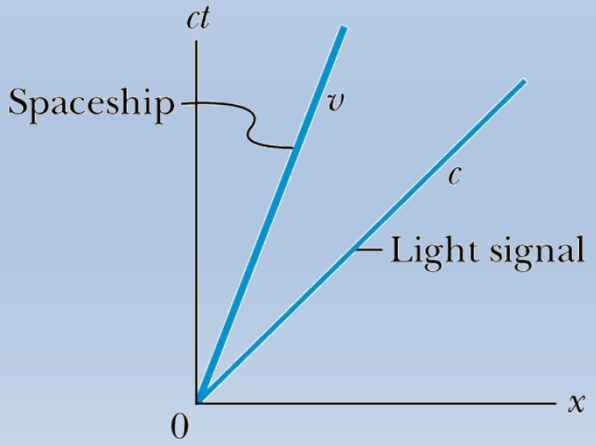
Space-time

When describing events in relativity, it is convenient to represent events on a **space-time** diagram at 4-dimensions: x, y, z, ct..

In this diagram one spatial coordinate *x,* to specify position, is used and instead of time *t*, *ct* is used as the other coordinate so that both coordinates will have dimensions of length.

Paths in the spacetime are called **worldliness** and points (A, B, events) are called **world points**





Particular worldliness are:

- ***Spaceship***, v = constant;
- ***Light signal***, v = c



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Spacetime Invariants

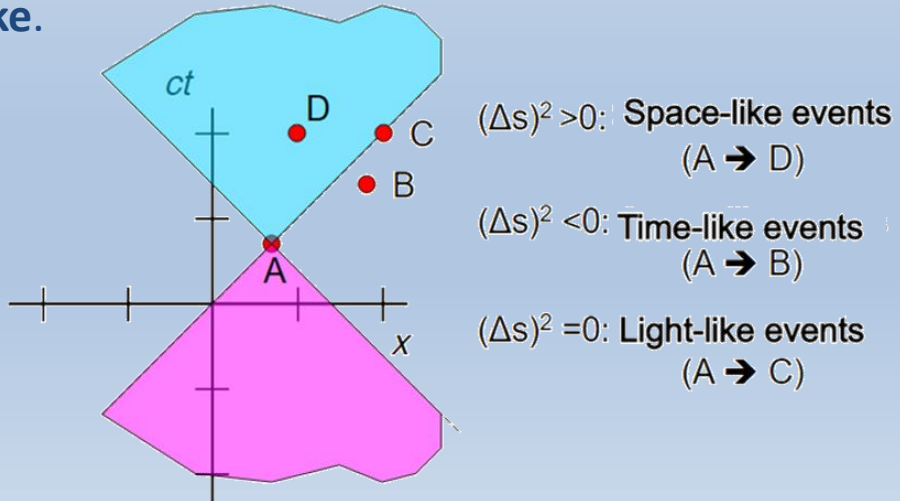
In this frame we observe elliptical geometry and we have an invariant:

$$\Delta s^2 = x^2+y^2+z^2-(ct)^2$$

We can demonstrate, using Lorentz Transformations, that: $\Delta s^2 = \Delta s'^2$

There are three possibilities for the invariant quantity Δs^2:

- $\Delta s^2 = 0$: $\Delta x^2 = c^2 \Delta t^2$, and the two events can be connected only by a light signal. The events are said to have a **lightlike** separation.
- $\Delta s^2 > 0$: $\Delta x^2 > c^2 \Delta t^2$, and no signal can travel fast enough to connect the two events. The events are not causally connected and are said to have a **spacelike** separation.
- $\Delta s^2 < 0$: $\Delta x^2 < c^2 \Delta t^2$, and the two events can be causally connected. The interval is said to be **timelike**.



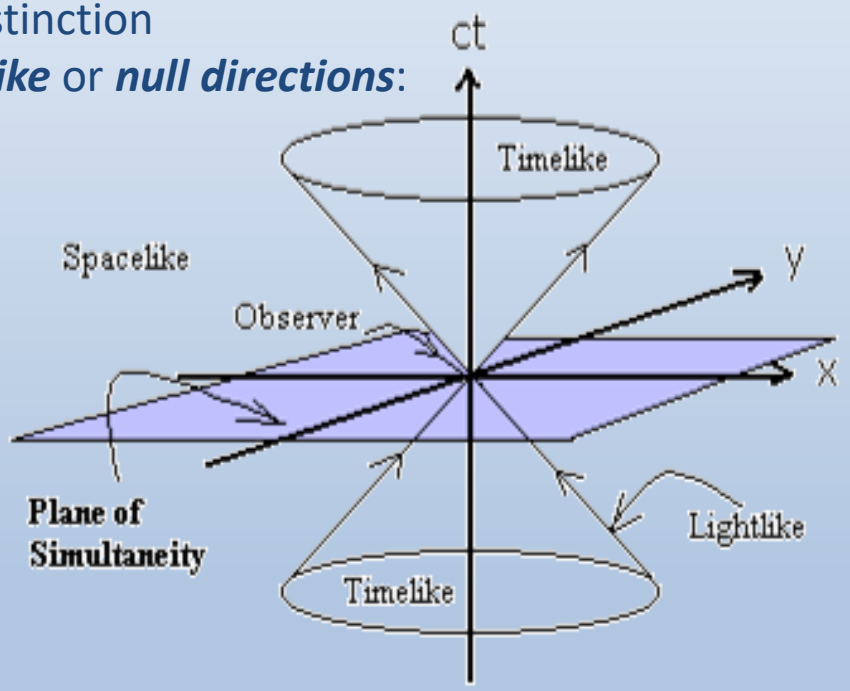


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Minkowski Chronotope

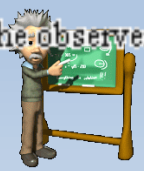
The spacetime is known as ***Minkowski chronotope***, where each point called "event" is a set of 4 coordinates. In this frame we can observe events in: the past, the present and the future. We can also make a distinction between **timelike**, **spacelike** and ***lightlike*** or ***null directions***:

- ***Timelike*** directions are those traced out over time by the ordinary objects around us;
- ***Spacelike*** direction "east" is something we can draw on a diagram;
- ***Lightlike*** are those traced out by light, in between the timelike and spacelike directions.



The plane of simultaneity is spacelike separated from the observer.

In this frame we observe the respect of the symmetries such as some ***intervals***, ***time dilation*** and ***length contraction***.

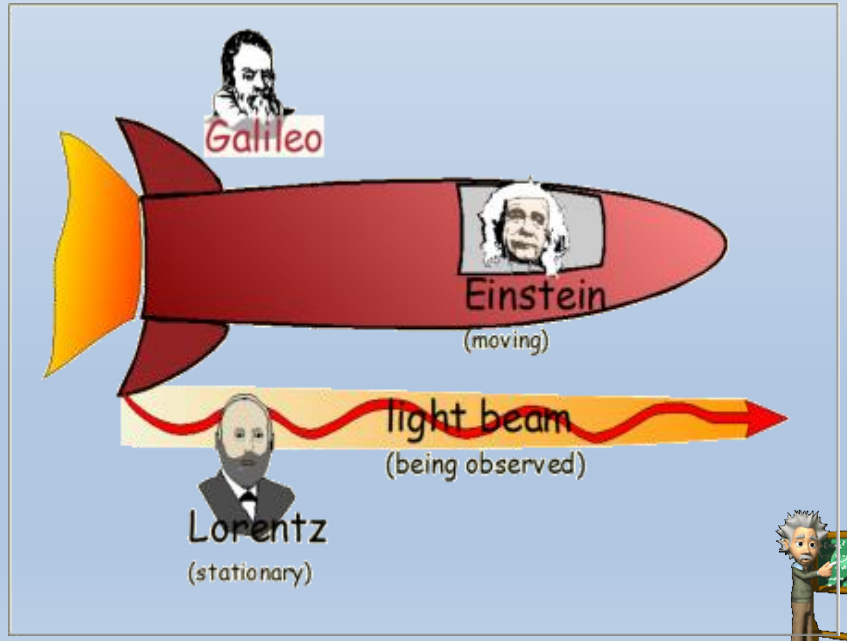


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Lorentz Transformations

Maxwell presupposed the existence of an absolute space in which were worth his equations but it was noted that the Maxwell equations were not formally invariant under Galilei's transformations from an Inertial Reference System to another. Einstein thought that the Lorentz Transformation equations describe properties of space and time itself. The term "Lorentz transformations" only refers to transformations between inertial frames, usually in the context of special relativity.

Historically, the transformations were the result of attempts by Lorentz and others to explain how the speed of light was observed to be independent of the reference frame, and to understand the symmetries of the laws of electromagnetism. The Lorentz transformation is in accordance with special relativity, but was derived before special relativity.



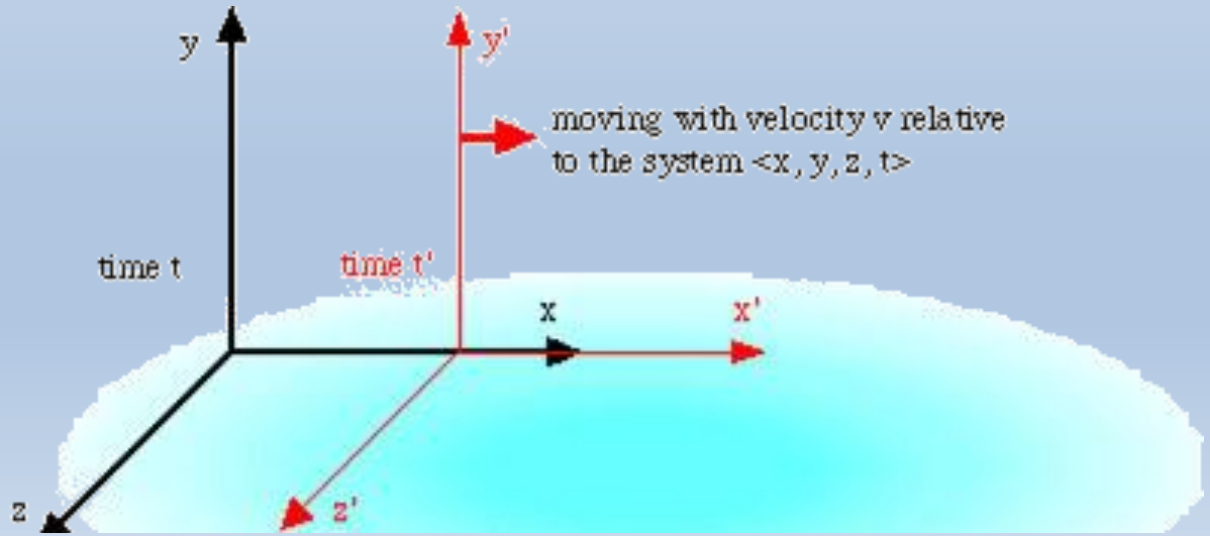
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Lorentz Transformations

Unlike the Galilean transformation, the Lorentz transformation involves a change of spatial distance and a change of time interval between two inertial systems. For a simple case where the new coordinate system is moving with constant velocity *v* along the x-axis of the old coordinate system , the transformation is as in the figure, where y' = y and z' = z.

1) If *v* << *c*, we see these equations reduce to the familiar Galilean transformation.
2) Space and time are now not separated.
3) For non-imaginary transformations, the frame velocity cannot exceed *c*.

$$\begin{cases} x' = \dfrac{x - vt}{\sqrt{1 - \dfrac{v^2}{c^2}}} \\ y' = y \\ z' = z \\ t' = \dfrac{t - \dfrac{vx}{c^2}}{\sqrt{1 - \dfrac{v^2}{c^2}}} \end{cases}$$





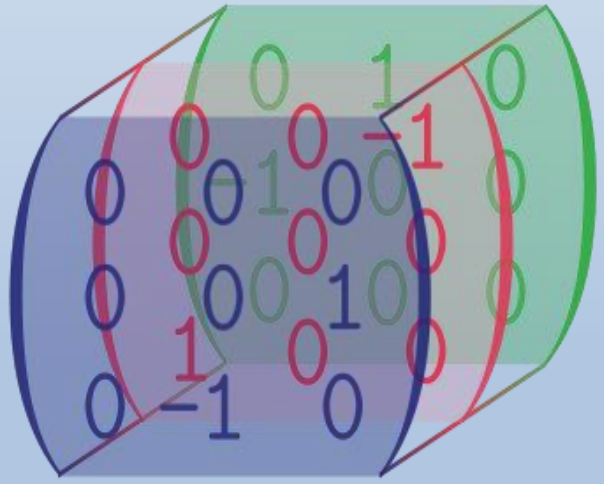
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Tensors

It was used by Albert Einstein to develop his theory of relativity. Respect to infinitesimal calculus, the tensor calculus allows you to present the physical equations in independent form by the choice of the coordinate system. By using tensors you can express the phenomena in objective form, to explain the laws of physics as combinations of the more profound laws, those of space-time.

A tensor can be described informally as a "matrix to more dimensions" containing arbitrary values. Examples of particular tensors are the scalar products and the vectors, such as the ***momentum-energy tensor***, that is defined in the context of the theory of relativity.

$$\epsilon_{ijk} =$$



In the figure, there is shown a three-dimensional matrix (a cube) containing $3^3 = 27$ numbers, which represents a tensor.

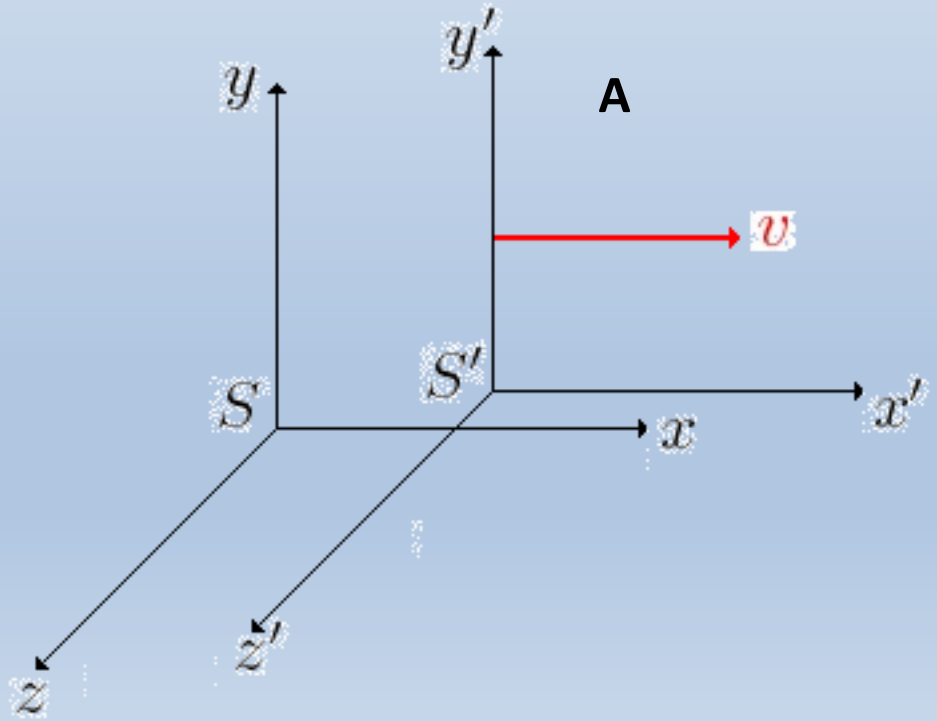


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Learning Check 4

Consider two frames of reference *S* and *S′* in standard configuration: *S′* moves along the +*x* axis of frame *S* with speed *v*. The origins of the two reference frames coincide at *t* = 0.

Let *v* = 0.80*c* and let "c·s" denote one light-second (the distance light in a vacuum travels in one second). Find the coordinates of the event, A in frame S′, knowing that the coordinates of this event in *S* are x_A= 1 c·s and t_A= 2 s



$$\begin{cases} x' = \gamma(x - \beta c) \\ y' = y \\ z' = z \\ t' = \gamma(t - \beta x/c) \end{cases}$$

$$\beta = v/c$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$



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Solution 4

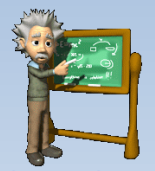
Lorentz Transformations

Given	Requests
$x_A = 1$ c·s $t_A = 2$ s	x'_A t'_A

Work

$$x'_A = \frac{x_A - vt_A}{\sqrt{1-\frac{v^2}{c^2}}}: \quad x'_A = \frac{1.0\ \text{c}\cdot\text{s} - (0.80c)(2.0\ \text{s})}{\sqrt{1-\frac{(0.80c)^2}{c^2}}}; \quad x'_A = \frac{1.0\ \text{c}\cdot\text{s} - 1.6\ \text{c}\cdot\text{s}}{0.60}; \quad x'_A = -1.0\ \text{c}\cdot\text{s}$$

$$t'_A = \frac{t_A - \frac{vx_A}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}: \quad t'_A = \frac{2.0\ \text{s} - \frac{(0.80c)(1.0\ \text{c}\cdot\text{s})}{c^2}}{\sqrt{1-\frac{(0.80c)^2}{c^2}}}; \quad t'_A = \frac{2.0\ \text{s} - 0.8\ \text{s}}{0.60}; \quad t'_A = 2.0\ \text{s}$$



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Special Relativity

Special relativity is the generally accepted and experimentally well-confirmed physical theory regarding the relationship between space and time. It implies a wide range of consequences, which have been experimentally verified, including length contraction, time dilation, relativistic mass, mass–energy equivalence, a universal speed limit and relativity of simultaneity.

The special relativity can be defined as the covariant formulation in terms of the Minkowski metric and Lorentz tensors; it is a theory of a fundamental symmetry principle of physics, namely that the laws of physics are invariant under Lorentz transformations and that they should therefore also be formulated in a way which makes this symmetry manifest.





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Einstein's Two Postulates

:

Accordingly the results of the Michelson-Morley experiment and with the belief that Maxwell's equations must be valid in all inertial frames, Einstein proposes the following postulates:

1) **The principle of relativity**: The laws of physics are the same in all inertial systems. There is no way to detect absolute motion, and no preferred inertial system exists.
2) **The constancy of the speed of light**: Observers in all inertial systems measure the same value for the speed of light in a vacuum.

It is believed that the theory of relativity makes, in physics, all relative destroying any absolute, such as the weather. Rather, it makes relating things that were considered absolute but introduces new absolute.

For example, the speed of light becomes, in this sense, a new absolute.





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Velocities composition

Taking differentials of the Lorentz transformation, relative velocities may be calculated:

$$dx = \gamma(dx' + v\,dt')$$
$$dy = dy'$$
$$dz = dz'$$
$$dt = \gamma[dt' + (v/c^2)\,dx']$$

defining velocities as: $u_x = dx/dt$, $u_y = dy/dt$, $u'_x = dx'/dt'$, etc. it is easily shown that:

$$u_x = \frac{dx}{dt} = \frac{\gamma(dx' + v\,dt')}{\gamma[dt' + (v/c^2)\,dx']} = \frac{u'_x + v}{1 + (v/c^2)u'_x}$$

With similar relations for u_y and u_z:

$$u_y = \frac{u'_y}{\gamma\left[1 + (v/c^2)u_x'\right]} \qquad u_z = \frac{u'_z}{\gamma\left[1 + (v/c^2)u_x'\right]}$$



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Lorentz velocity transformations

In addition to the previous relations we can consider two limit cases:

- for u'_x << c and v << c, we obtain the classical composition:

$$u_x = \frac{u'_x + v}{1+0} \qquad \rightarrow \qquad u_x = u'_x + v$$

- for u'_x = c we, obtain the limit velocity of the light:

$$u_x = \frac{c + v}{1 + \left(\frac{v}{c}\right)} = c$$

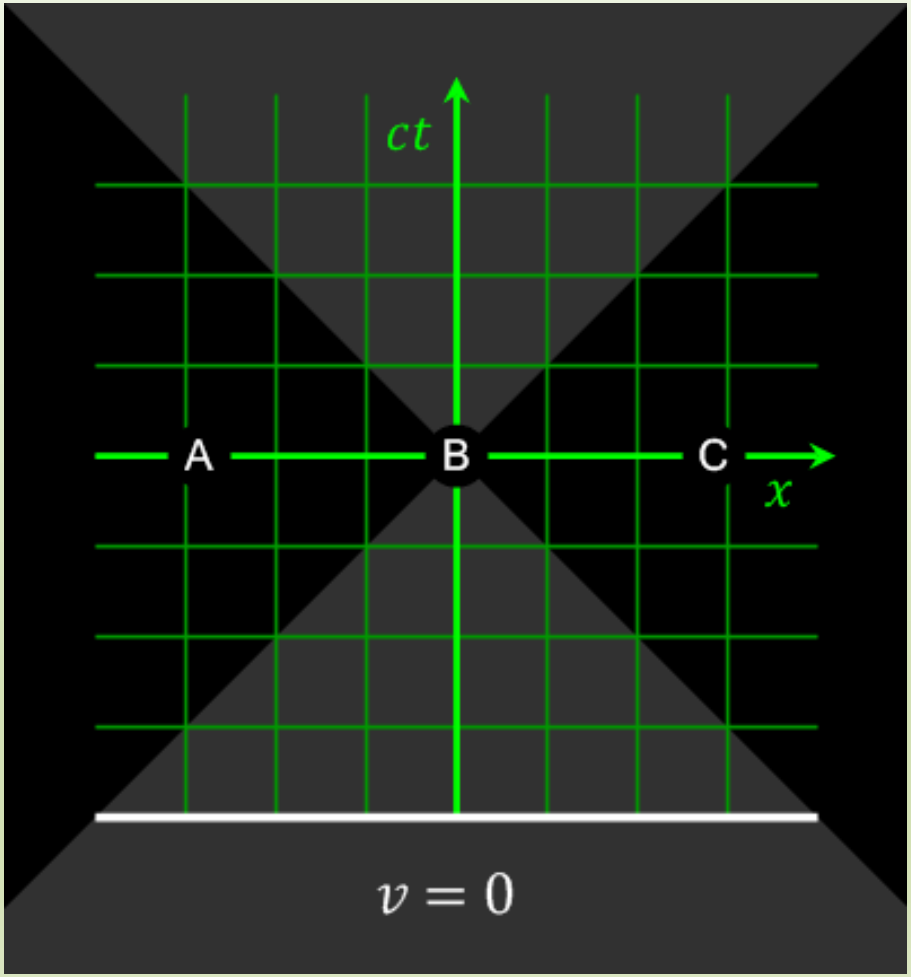




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The Problem of Simultaneity

Einstein realized that each system must have its own observers with their own clocks and meter sticks. Two events that are simultaneous in one reference frame are not necessarily simultaneous in another reference frame moving with respect to the first frame. This suggests that each coordinate system has its own observers with "clocks" that are synchronized…





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Time Dilation and Length Contraction

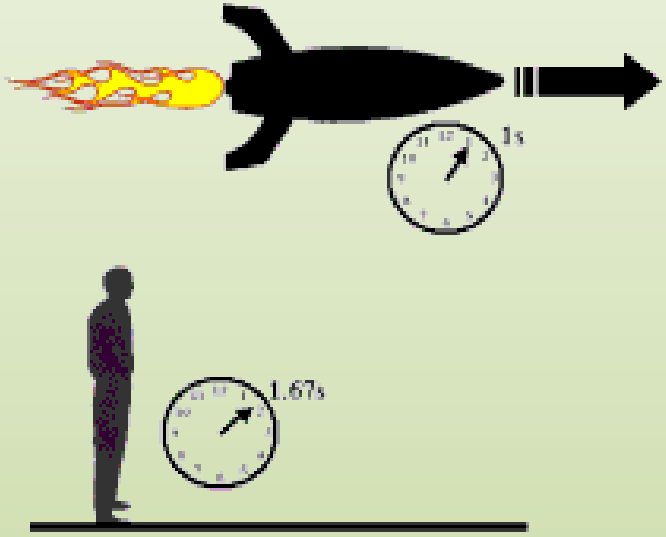
In trying to answer the question "what is relativity?", we have indirectly 'stumbled upon' the phenomena of ***time dilation and length contraction*** due to velocity. The constancy (or invariance) of the space-time interval is underlying to relativistic time dilation and length contraction. Consequences of the Lorentz Transformation:

- **Time Dilation**: Clocks in a frame S' run slow with respect to stationary clocks in a frame S.

  $\Delta t = \Delta t' \sqrt{1 - v^2/c^2} \rightarrow \Delta t < \Delta t'$
- **Length Contraction**: Lengths ***L'*** in a frame S' are contracted with respect to the same lengths stationary ***L*** in a frame S.

  $\Delta l' = \Delta l \sqrt{1 - v^2/c^2} \rightarrow \Delta l > \Delta l'$

  $L' = L \sqrt{1 - v^2/c^2}$



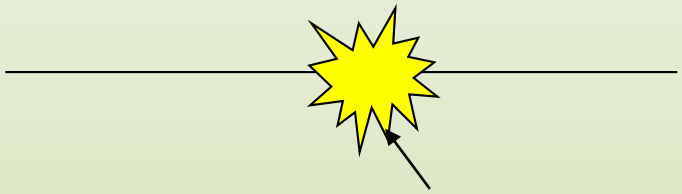


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Time Dilation

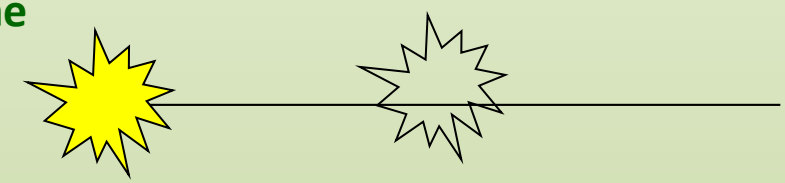
To understand time dilation the idea of proper time must be understood:

- The ***proper time*** between two events is the amount of time between those events as measured by an inertial reference frame in which those events occur at the same place.



Same location

- **Not Proper Time**



Beginning and ending of the event occur at different positions

Time dilation is a symmetrical phenomenon, because in the relative motion each observer associates this phenomenon to the other observer



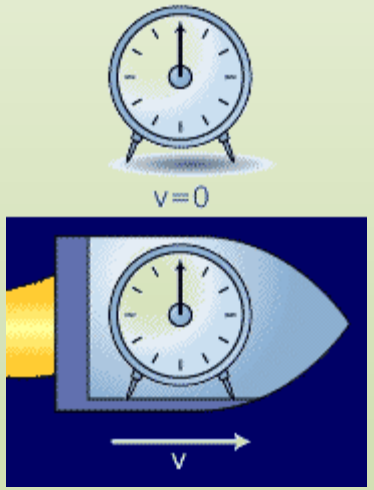
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Time Dilation

Using the Lorentz transformations we can demonstrate ***time dilation***:

$$\Delta t = t_2 - t_1 = \frac{t_2' + \frac{v}{c^2}x'}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{t_1' + \frac{v}{c^2}x'}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{t_2' - t_1'}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow$$

$$\Delta t = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}}.$$



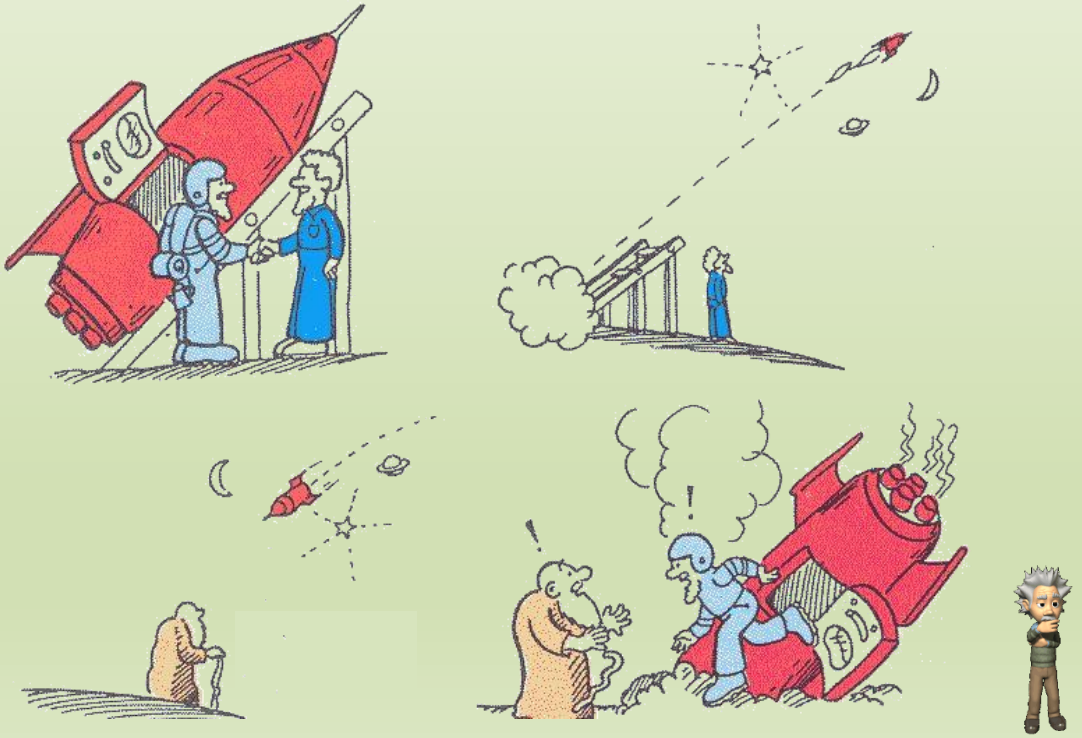


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Twin paradox

A consequence of time dilation is the ***twin paradox***: if we consider two identical clocks, one of which departs and the later returns, having travelled some distance in the intervening time at great speed. The clock which went on the voyage evidences less elapsed time.

For example you can imagine two twins each 20 years old.
One departs on a space voyage, traveling at 0.8 c, to a star 20 light years away. When he returns, he is 50 years old, while the other twin is 70 years old.

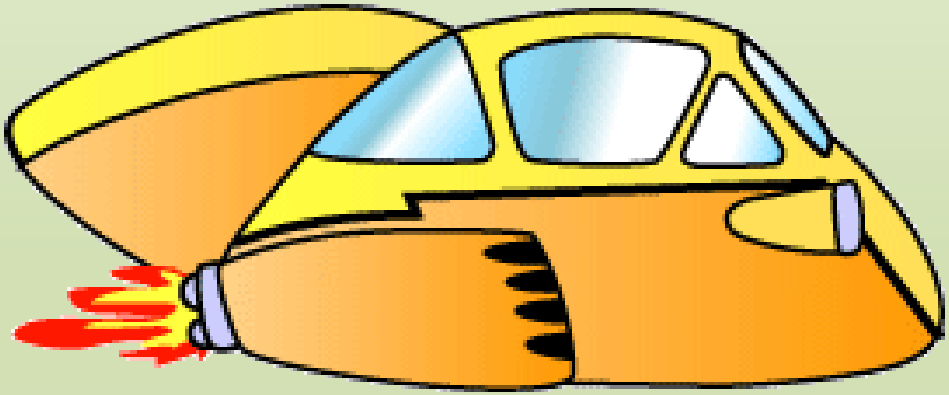


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Length Contraction

To understand *length contraction* the idea of **proper length** must be understood:

- Let an observer in each system K and K' have a meter stick at rest in their own system such that each measure the same length at rest.
- The length as measured at rest is called the **proper length**





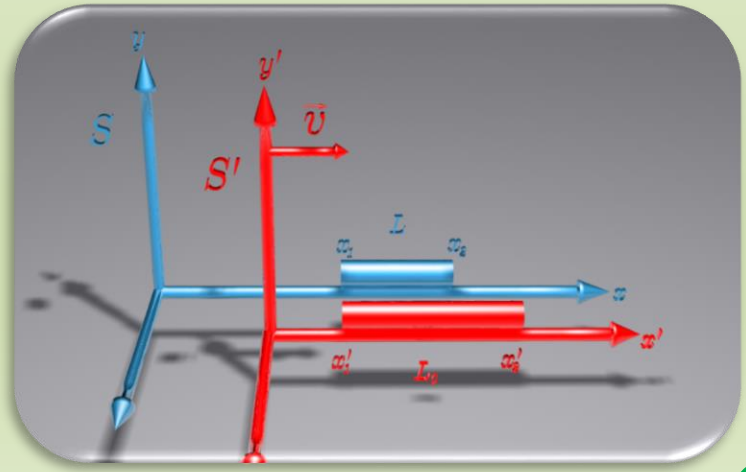
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Length Contraction

Using the Lorentz transformations and noting that in S' $t'_1 = t'_2$, we can demonstrate ***length contraction***:

$$l = x_2 - x_1 = \frac{x'_2 + vt'_2}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{x'_1 + vt'_1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{x'_2 - x'_1}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{v(t'_2 - t'_1)}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{x'_2 - x'_1}{\sqrt{1-\frac{v^2}{c^2}}} \rightarrow l = \frac{l'}{\sqrt{1-\frac{v^2}{c^2}}}$$

The length contraction takes place only in the direction of motion, there is no contraction in the transverse direction.
It is also a symmetrical phenomenon, because in the relative motion each observer associates this phenomenon to the other observer

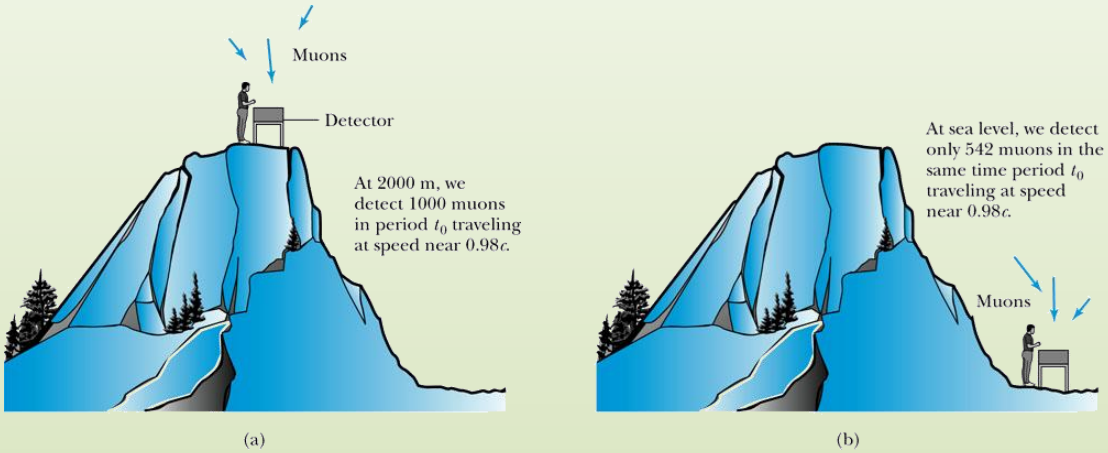




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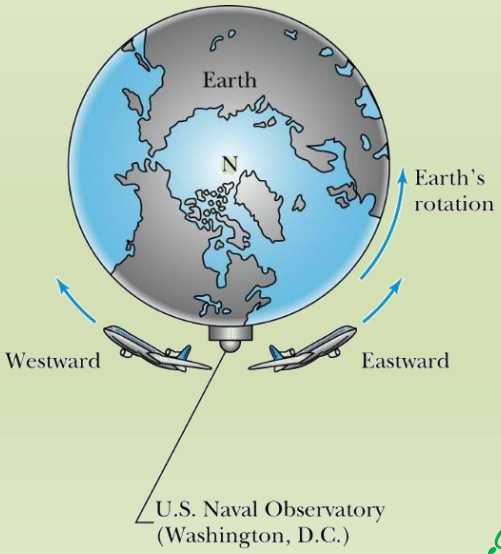
Experimental Verification

Time Dilation and Muon Decay
The number of muons detected with speeds near 0.98*c* is much different (a) on top of a mountain than (b) at sea level, because of the muon's decay. The experimental result agrees with our time dilation equation.



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Atomic Clock Measurement
Two airplanes took off (at different times) from Washington, D.C., where the U.S. Naval Observatory is located. The airplanes traveled east and west around Earth as it rotated. Atomic clocks on the airplanes were compared with similar clocks kept at the observatory to show that the moving clocks in the airplanes ran slower.



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Learning Check 5

A. Inertial systems are accelerated true/false

B. Light velocity c is constant true/false

C. In the velocity composition there is time true/false

D. The time dilatation is obtained by Lorentz true/false

E. The length contraction is only at rest true/false

F. S. Relativity has experimental verification true/false



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Solution 5

A. Inertial systems are accelerated true/false

B. Light velocity c is constant true/false

C. In the velocity composition there is time true/false

D. The time dilatation is obtained by Lorentz true/false

E. The length contraction is only at rest true/false

F. S. Relativity has experimental verification true/false



R. Pancari

Mass in special relativity

The word "mass" is given two meanings in special relativity: one m_0 ("rest mass" or "invariant mass") is an invariant quantity which is the same for all observers in all reference frames; the other m ("relativistic mass") is dependent on the velocity of the observer.

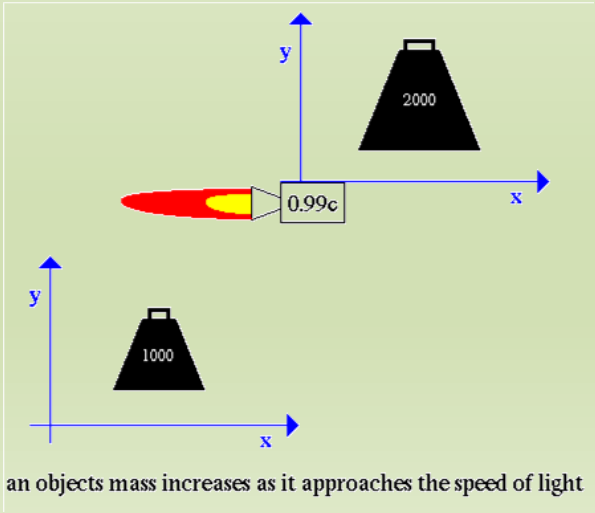
$$m_{relativistic} = \frac{m_{rest}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$m_{relativistic} = \frac{m_{rest}}{\sqrt{1 - \frac{0^2}{c^2}}}$$

$$m_{relativistic} = m_{rest}$$

Mass in special relativity incorporates also the general understandings from the concept of mass–energy equivalence.

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$



an objects mass increases as it approaches the speed of light



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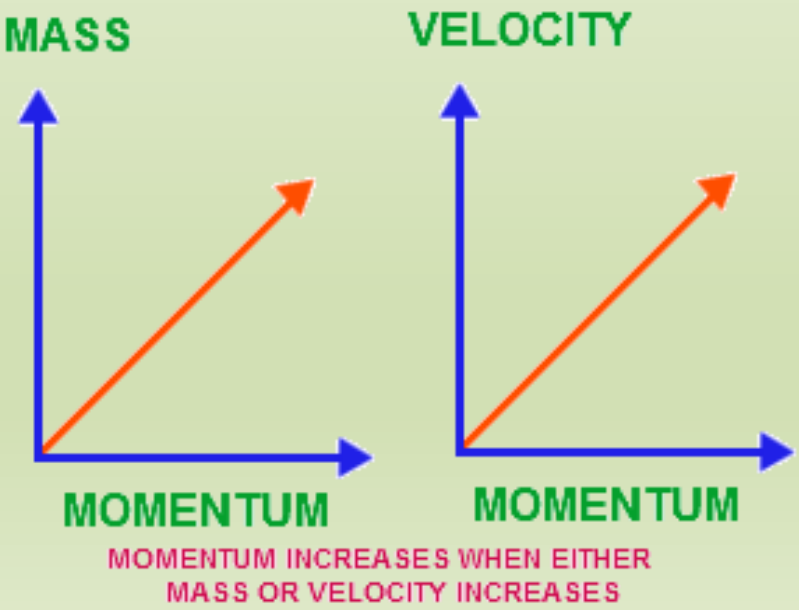
Relativistic Momentum

Rather than abandon the conservation of linear momentum, let us look for a modification of the definition of linear momentum that preserves both it and Newton's second law.

To do so requires reexamining mass to conclude that:

$$\vec{p} = m\frac{d\vec{r}}{dt}\gamma \quad \rightarrow \quad p = \gamma mu = \frac{mu}{\sqrt{1-u^2/c^2}}$$

Relativistic momentum



The mass in Equation is the relativistic mass $m = \gamma m_0$. In this manner the classical form of momentum, m, is retained. The mass is then imagined to increase at high speeds.



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Relativistic Energy

Due to the new idea of relativistic mass, we must now redefine the concepts of work and energy.
Therefore, we modify Newton's second law to include our new definition of linear momentum, and force becomes:

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(\gamma m\vec{u}) = \frac{d}{dt}\left(\frac{m\vec{u}}{\sqrt{1-u^2/c^2}}\right)$$

For simplicity, let the particle start from rest under the influence of the force and calculate the kinetic energy K after the work is done.

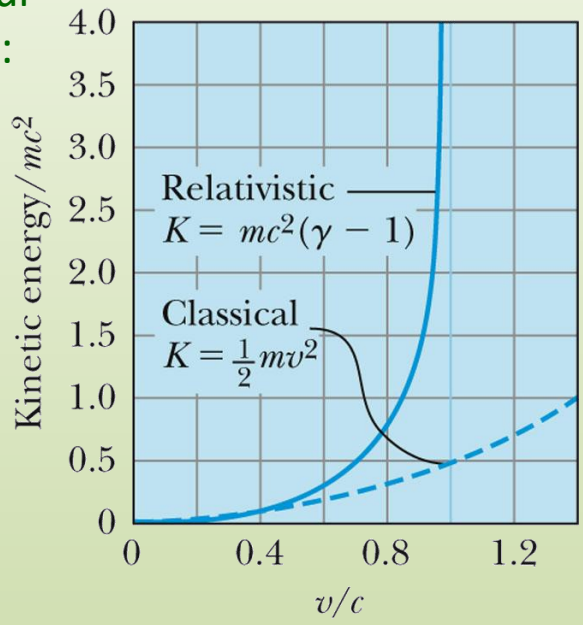
$$W = K = \int \frac{d}{dt}(\gamma m\vec{u})\cdot\vec{u}\ dt$$

The limits of integration are from an initial value of 0 to a final value of γu.

$$K = m\int_0^{\gamma u} u\ d(\gamma u)$$

The integral in Equation (2.57) is straightforward if done by the method of integration by parts. The result, called the relativistic kinetic energy, is:

$$K = \gamma mc^2 - mc^2 = mc^2\left(\frac{1}{\sqrt{1-u^2/c^2}} - 1\right) = mc^2(\gamma - 1)$$



For speeds $u << c$, we expand γ in a binomial series and obtain the classical formula.



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Total Energy and Rest Energy

We rewrite Equation in the form

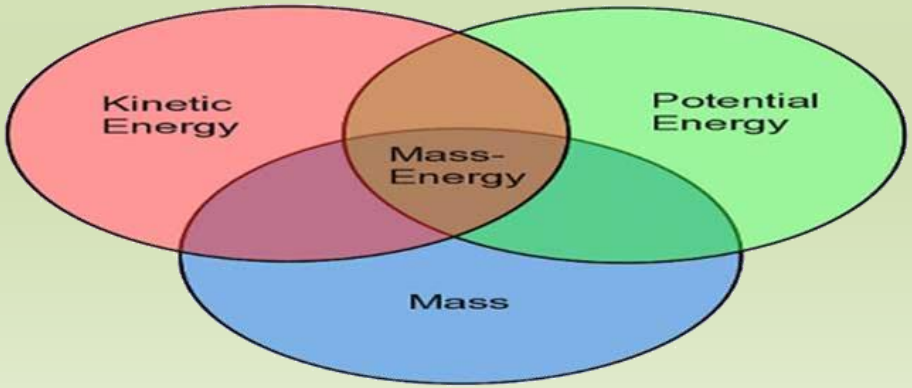
$$\gamma mc^2 = \frac{mc^2}{\sqrt{1-u^2/c^2}} = K + mc^2$$

The term mc^2 is called the rest energy and is denoted by E_0.

$$E_0 = mc^2$$

This leaves the sum of the kinetic energy and rest energy to be interpreted as the total energy of the particle. The total energy is denoted by E and is given by

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-u^2/c^2}} = \frac{E_0}{\sqrt{1-u^2/c^2}} = K + E_0$$





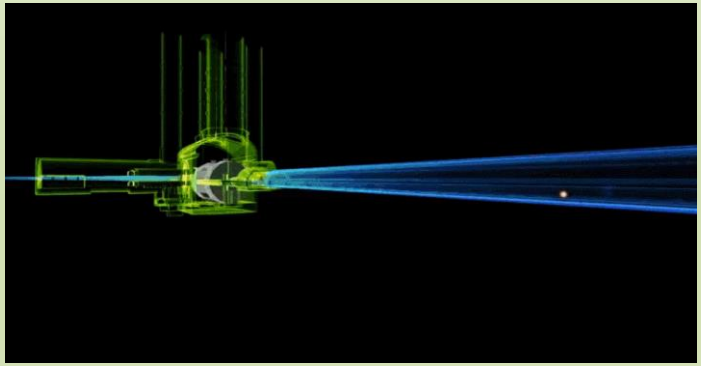
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Mass–energy equivalence

Since special relativity demonstrates that space and time are variable concepts, then velocity (which is space divided by time) becomes a variable as well. If velocity changes from reference frame to reference frame, then concepts that involve velocity must also be relative. One such concept is momentum, motion energy. Momentum, as defined by Newton, cannot be conserved from frame to frame under special relativity. A new parameter had to be defined, called relativistic momentum, which is conserved, but only if the mass of the object is added to the momentum equation.

This has a big impact on classical physics because it means there is an equivalence between mass and energy, summarized by the famous Einstein equation: $E = mc^2$

The implications of this was not realized for many years. For example in atomic processes in accelerator tubes, when an object is accelerated close to the speed of light, relativistic effects begin to dominate. In particular, adding more energy to an object will not make it go faster since the speed of light is the limit. The energy has to go somewhere, so it is added to the mass of the object, as observed from the rest frame.

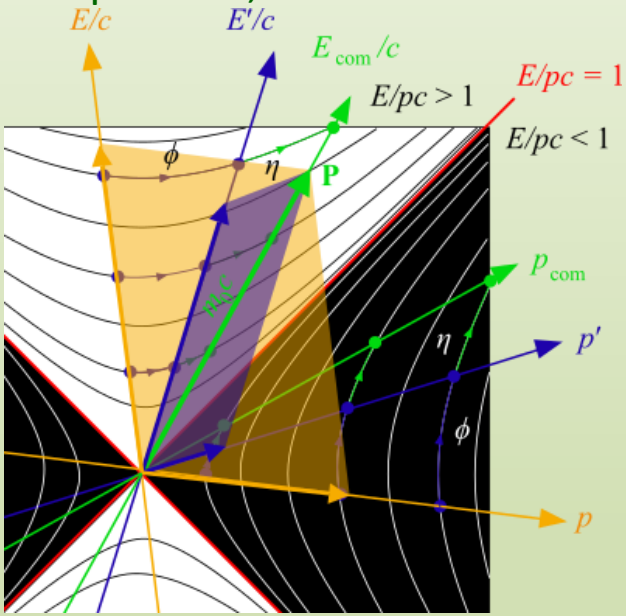




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Momentum-energy tensor

The ***momentum-energy tensor*** is a tensor defined in the context of the theory of relativity. It describes the flow of energy and momentum associated to a field. The spatial components of the tensor are the classic three-dimensional impulse components, while the time component is the energy divided by the speed of light.



In the figure the green frame is the center of momentum frame for the object with energy equal to the rest energy.

In special relativity, ***four-momentum*** is the generalization of the classical three-dimensional momentum to four-dimensional space-time. Momentum is a vector in three dimensions; similarly four-momentum is a four-vector in space time, always tangent to the trajectory, or world-line, of a particle, its matrix form is:

$$\langle \mathbf{P}, \mathbf{P} \rangle = P^\alpha \eta_{\alpha\beta} P^\beta = \begin{pmatrix} \frac{E}{c} & p_x & p_y & p_z \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{E}{c} \\ p_x \\ p_y \\ p_z \end{pmatrix} = -\left(\frac{E}{c}\right)^2 + p^2 ,$$

where

$$-(m_0 c)^2 = -\left(\frac{E}{c}\right)^2 + p^2 .$$



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Kinetic Energy - Momentum

If we square the formula of momentum, multiply by c^2, and rearrange the result we obtain:

$$p^2c^2 = \gamma^2 m^2 u^2 c^2 = \gamma^2 m^2 c^4 \left(\frac{u^2}{c^2}\right) = \gamma^2 m^2 c^4 \beta^2$$

We use Equation for β^2 and find

$$p^2c^2 = \gamma^2 m^2 c^4 \left(1 - \frac{1}{\gamma^2}\right) = \gamma^2 m^2 c^4 - m^2 c^4$$

The first term on the right-hand side is just E^2, and the second term is E_0^2. The last equation becomes

$$p^2c^2 = E^2 - E_0^2$$

We rearrange this last equation to find the result we are seeking, a relation between energy and momentum.

$$E^2 = p^2c^2 + E_0^2 \qquad \text{or} \qquad E^2 = p^2c^2 + m^2c^4$$

Equation is a useful result to relate the total energy of a particle with its momentum. The quantities ($E^2 - p^2c^2$) and m are invariant quantities. Note that when a particle's velocity is zero and it has no momentum, Equation correctly gives E_0 as the particle's total energy.



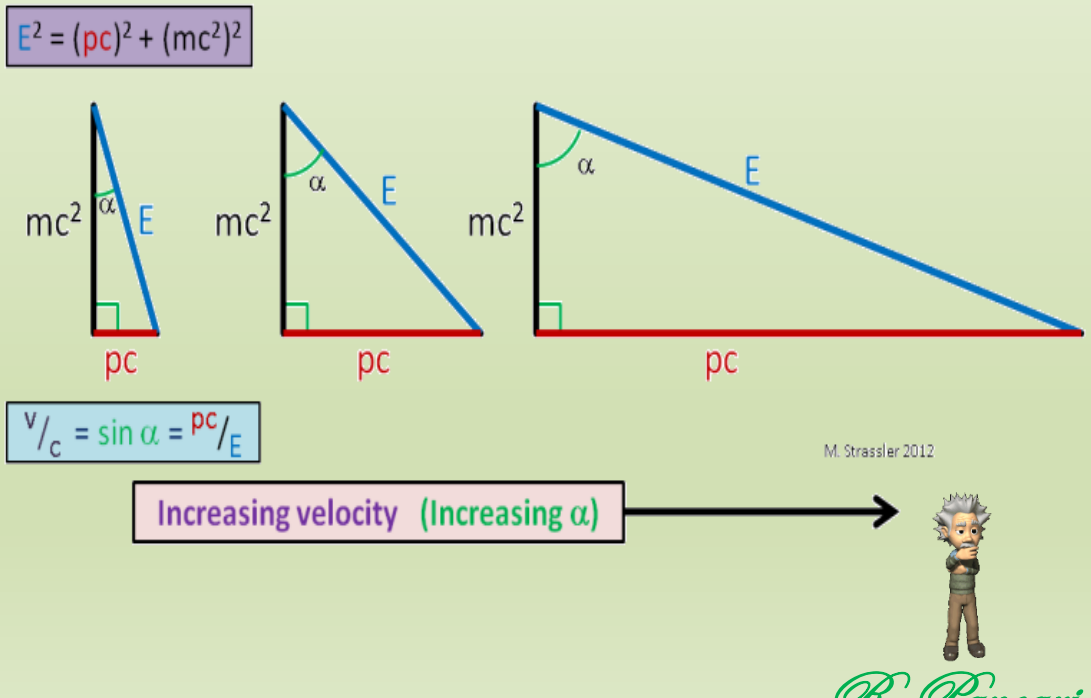
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The invariant Energy - Momentum

In classical physics the energy ***E*** and the momentum ***p*** are constant separately, in special relativity the quantity Energy-Momentum m^2c^4 is constant in all frames:

$$E^2 - p^2c^2 = m^2c^4$$

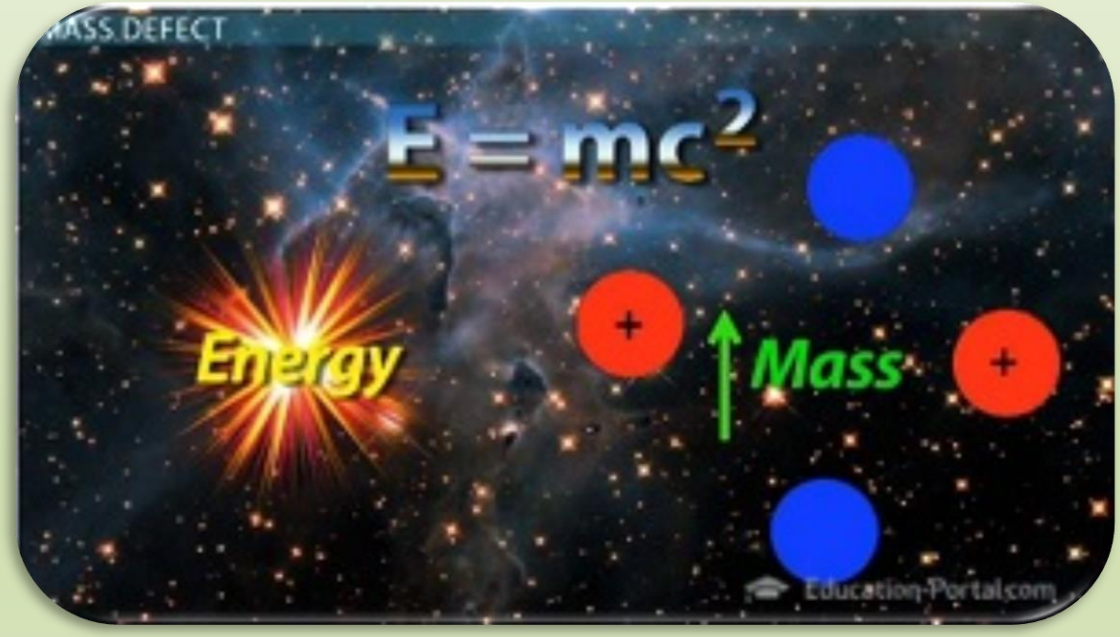
For an object moving according to Einstein's relativity, the relation between energy, momentum and mass is analogous to the Pythagorean relationship between the hypotenuse of a right-angle triangle and its two sides. The velocity of the particle, divided by c, is the ratio of the length of the horizontal line to the length of the hypotenuse; this is also the sine of the angle α



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Learning Check 6

If 1 kg of mass is converted to energy and used to accelerate a second 1 kg mass, what would be the final velocity?





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Solution 6

Mass-Energy equivalence

Given	Requests
m_1= 1 kg m_2= 1 kg m_1= E	v_2

Work

$$\boldsymbol{E = 2mc^2}$$

$$\boldsymbol{E = \gamma mc^2 \rightarrow \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \rightarrow \gamma = 2 \rightarrow v = \frac{\sqrt{3}}{2}c \rightarrow v \approx 0,866c}$$



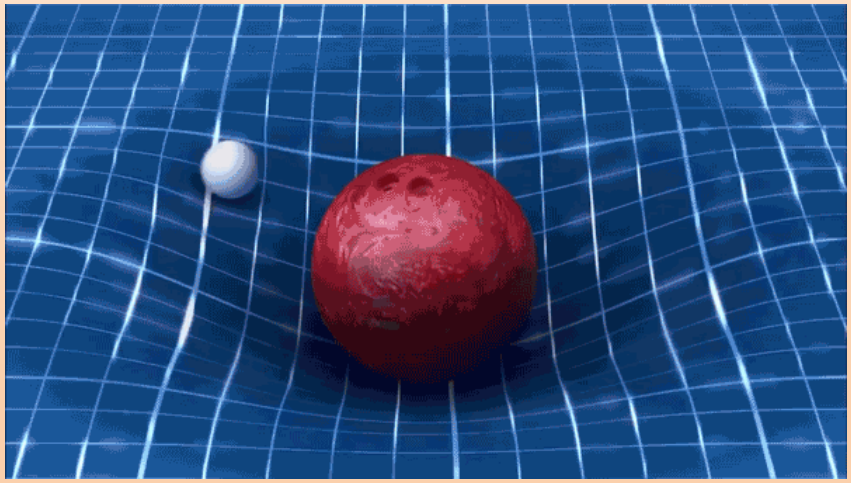
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General Relativity

After the formulation of the Special Relativity, Einstein had two problems:

- the Physics should be based on a more general principle of relativity for which the laws should also be valid in the non-inertial reference frame;
- the difficulty of explaining the gravitational theory with special relativity.

The resolution of these problems led to the construction of the General Relativity Theory, that is a "generalized" and enhanced version of Special Relativity, in which Einstein formulated the ***equality between Inertial and Gravitational Mass*** and the ***Principle of Equivalence.***
He used some mathematical tools such as the geodesics and the tensors to describe all the properties of Gravitation.





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The Equality of Inertial and Gravitational Mass

According to Newton's law of motion, we have: $F = m_i a$ (m_i = inertial mass), where the "inertial mass" is a characteristic constant of the accelerated body. If now gravitation is the cause of the acceleration, we then have: $F = m_g g$ (m_g = gravitational mass), where the "gravitational mass" is likewise a characteristic constant for the body.

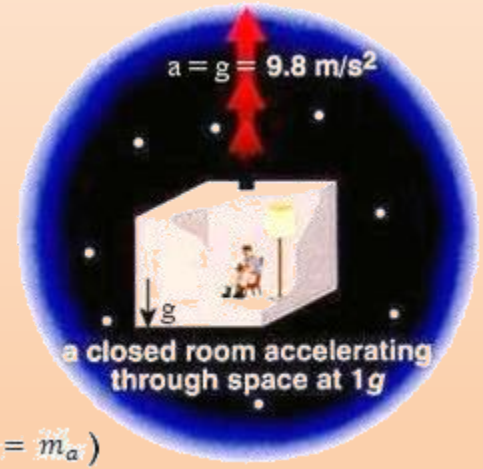
From these two relations follows:

$$a = \frac{m_g}{m_i} g$$

As we find from experience, the acceleration is to be independent of the nature and the condition of the body and always the same for a given gravitational field, then the ratio of the gravitational to the inertial mass must likewise be the same for all bodies. By a suitable choice of units we can thus make this ratio equal to unity. We then have the following law:

The gravitational mass of a body is equivalent to its inertial mass.





Principle of Equivalence (if $m_g = m_a$)

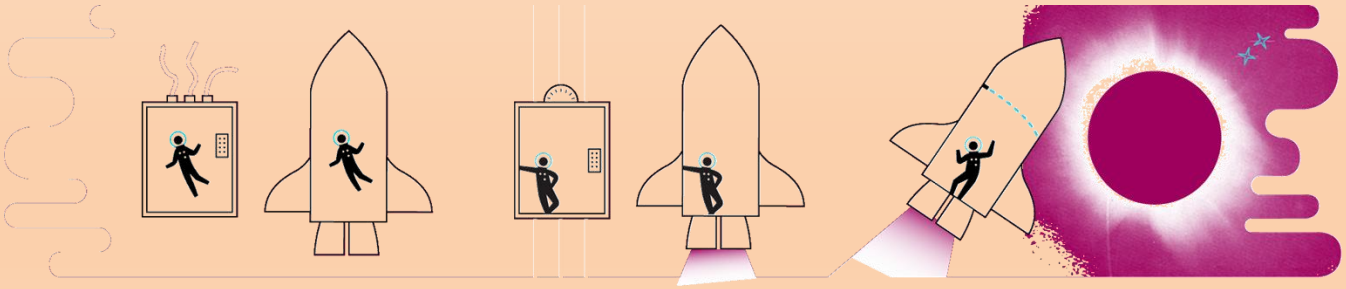
The same quality of a body manifests itself according to circumstances as "inertial " or as " gravitational".



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GRAVITY AND ACCELERATION

The equality between the masses shows that the same quality of a body manifests itself according to circumstances as "inertial " or as " gravitational". So Einstein realized that there was no difference in the force of gravity and the force of acceleration. For example, someone in a space ship without windows cannot tell whether the ship is a rest on Earth or is accelerating through outer-space. The net effect (the person's feet pushing against the floor of the spaceship) is identical. If gravity has characteristics of motion, then strong gravitational fields must make matter behave similar to high velocities. In other words, he postulated that no experiment can locally distinguish between a uniform gravitational field and a uniform acceleration.



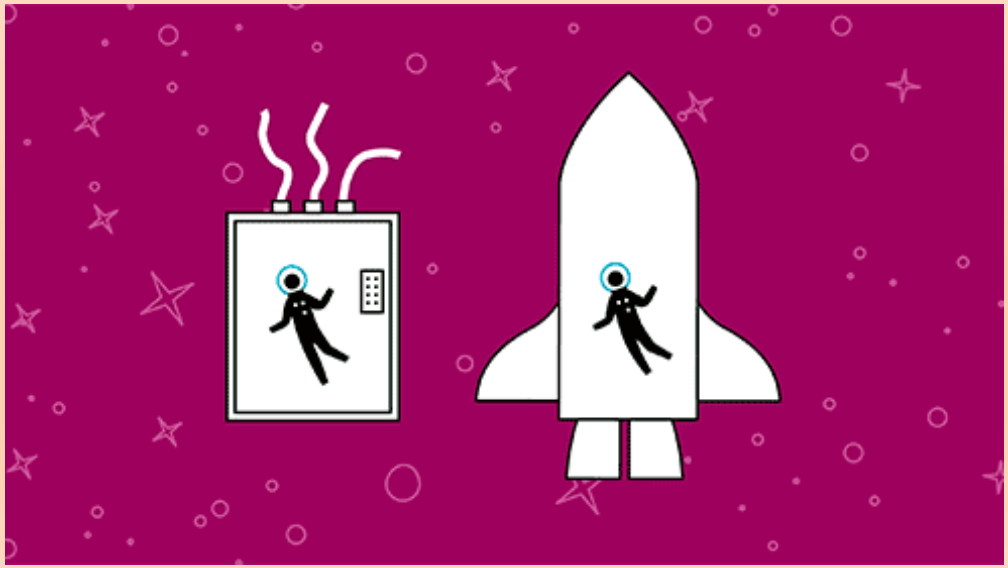


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The Principle of Equivalence

This Principle describes gravitation and acceleration as different perspectives of the same thing, and it was originally stated by Einstein in 1907 as:

We shall therefore assume the complete physical equivalence of a gravitational field and the corresponding acceleration of the reference frame. This assumption extends the principle of relativity to the case of uniformly accelerated motion of the reference frame.





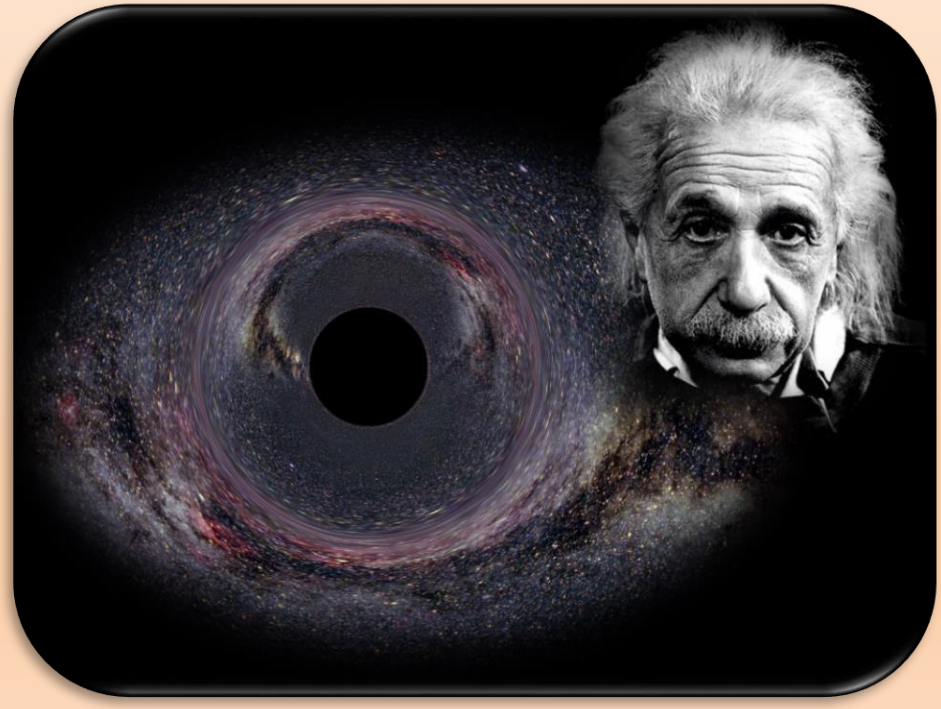
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General Relativity Principle

Whit the Principle of Equivalence Einstein postulated that no experiment can locally distinguish between a uniform gravitational field and a uniform acceleration, so he came to the formulation of the *Principle of General Relativity:*

The laws of physics take the same form respect to all the reference systems

GR is a theory of gravitation that supersedes Newton's Law of Universal Gravitation using the warping of spacetime by mass to explain gravitational attraction instead of the idea of "forces".
Essentially, massive bodies warp and curve their local spacetime





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Learning Check 7

A. Which theory does describe the General Relativity?

1) gravitation 2) dynamics

B. What is the result of the ratio between m_i and m_g?

1) ~ 1 2) ~ 0

C. In GR does exist a privileged reference frame?

1) yes 2) no



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Solution 7

A. Which theory does describe the General Relativity?

1) gravitation

B. What is the result of the ratio between m_i and m_g?

1) ~ 1

C. In GR does exist a privileged reference frame?

2) no



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Gravity as Curvature

Einstein Field Equations

Einstein said that gravity can be looked at as curvature in space-time and not as a force that is acting between bodies. So the Gravity field is not a force field but curvature of time-space caused by the presence of mass and energy, where the source of curvature is the stress–energy tensor, representing matter.

In GR ***geodesic*** generalizes the notion of a "straight line" to curved spacetime. Importantly, the world line of a particle free from all external, non-gravitational force, is a particular type of geodesic. In other words, a freely moving or falling particle always moves along a geodesic. A geodesic between two events can be described as the curve joining those two events which has a stationary interval (along the curve varies minimally among curves that are nearby to the geodesic).





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Einstein Field Equations

The Einstein field equations (EFE) is the set of 10 equations in Albert Einstein's general theory of relativity that describes the fundamental interaction of gravitation as a result of spacetime being curved by mass and energy. First published by Einstein in 1915 as a tensor equation, the EFE equate local spacetime curvature (expressed by the Einstein tensor) with the local energy and momentum within that spacetime (expressed by the stress–energy tensor)

Their demonstration is complex, the EFE are used to determine the metric tensor. The stress–energy tensor is the source of the gravitational field in the Einstein field equations of general relativity, just as mass density is the source of such a field in Newtonian gravity.

$$G_{\mu\nu} + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ = Einstein tensor

$R_{\mu\nu}$ = Ricci curvature tensor

R = Scalar curvature

$g_{\mu\nu}$ = Metric tensor

Λ = Cosmological constant

G = Newton's gravitational constant

c = Speed of light

$T_{\mu\nu}$ = Stress-energy tensor

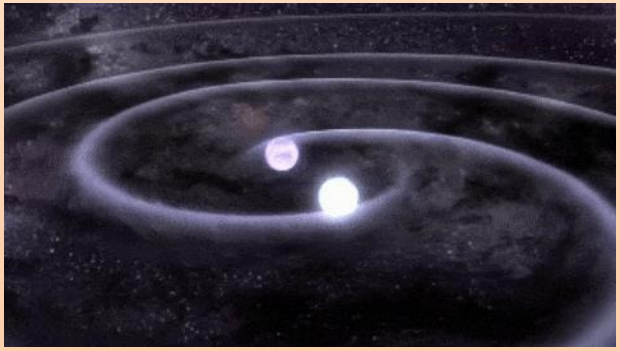


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Gravitational waves

Similar to how the motion of electrical charges generate waves in magnetic and electric fields, so too would it be with the gravitational field. Gravitational Waves are predicted by general relativity to be the oscillations of spacetime, propagating at the speed of light, alternating compressions and rarefactions of spacetime, but on a very small scale (as little as a factor of 10-21 even for a very strong source). GWs are produced from the acceleration of massive objects, they act by changing the distance between widely separated masses, or by changing the shapes of the objects they pass through, as they stretch along one axis and squeeze along the other.

They are described as a tensor perturbation to the metric of spacetime. At lowest order, GWs come from the time-changing quadrupole moment of mass and energy. The quadrupole formula reveals that gravitational waves are difficult to produce and that gravitational interactions are extremely weak. Gravitational waves have not yet been detected directly, but we believe that they have been detected indirectly by radio astronomers in the binary pulsar system.



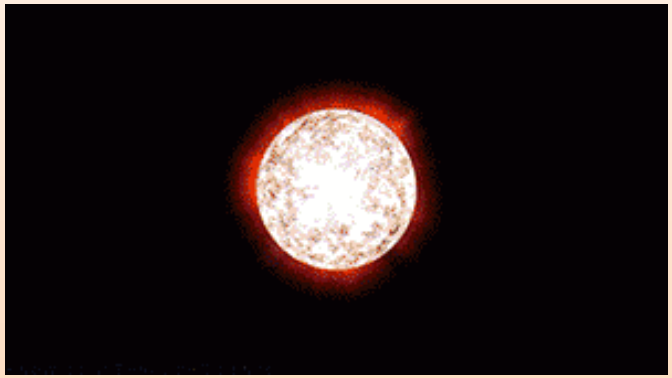
The existence of GW was confirmed experimentally on September 24, 2015 and announced on February 11, 2016 by the Advanced LIGO detector team, that observed simultaneously in Louisiana and Washington GW produced by a gigantic clash of two blacks holes distant 1,3 billions light years from the earth



R. Pancani

Black Holes

Predicted sources of strong gravitational waves in the Galaxy are also ***black holes***. Objects whose gravitational fields are too strong for light to escape were first considered in the 18th century by Michell and Laplace. The first modern solution of general relativity that would characterize a black hole was found by Karl Schwarzschild in 1916, although its interpretation as a region of space from which nothing can escape was first published by David Finkelstein in 1958.





The Einstein field equations predict that the curvature of space-time can increase endlessly around a star where contracting in the final phase of its existence, it reaches a sufficiently high density. If the concentration of stellar matter increases to a critical value its gravity becomes so high that it can not be balanced and arrives to the gravitational collapse that transforms the star into a black hole which has such a high gravity that attracts mass and light.



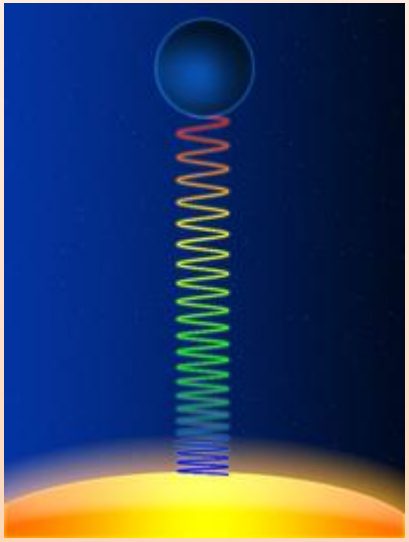
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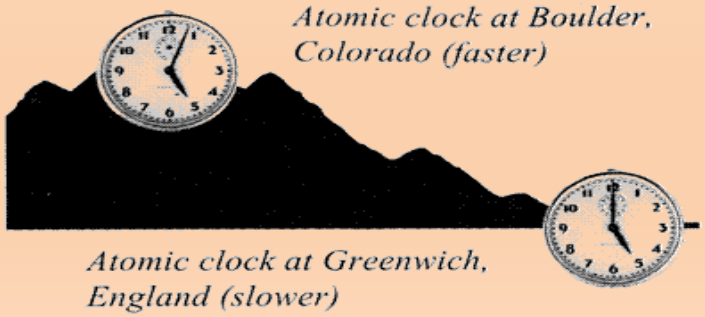
http://bestanimations.com/Earth&Space/Earth&Space3.html

Experimental Verification

Gravitational Redshift

In astrophysics, gravitational redshift or ***Einstein shift*** is the process by which electromagnetic radiation originating from a source that is in a gravitational field is reduced in frequency, or redshifted, when observed in a region at a higher gravitational potential. This is a direct result of gravitational time dilation. If applied to optical wavelengths, this manifests itself as a change in the colour, since frequency and wavelength are inversely proportional, this is equivalent to saying that the frequency of the light is reduced towards the red part of the light spectrum, giving this phenomenon the name redshift. Many astrophysical experiments verified the phenomenon.





Gravitational Time Dilation: *The rate at which an atomic clock records time is diminished as gravity increases.*

A consequence is the ***atomic clocks slowing down***, the pace of which is influenced by gravity, a high altitude clock shows the time faster than an identical one placed at sea level where gravity is more intense, numerous experiments have confirmed the slowdown.

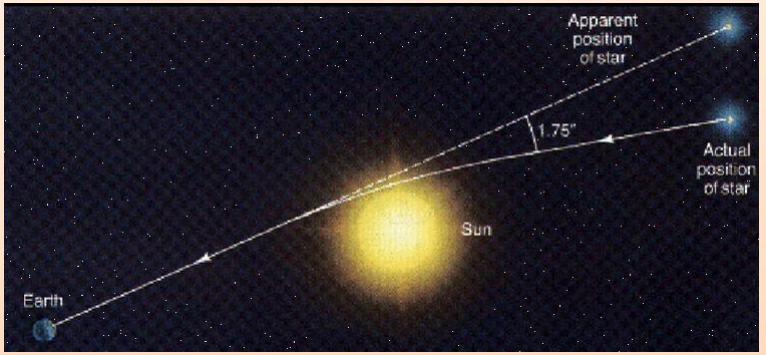


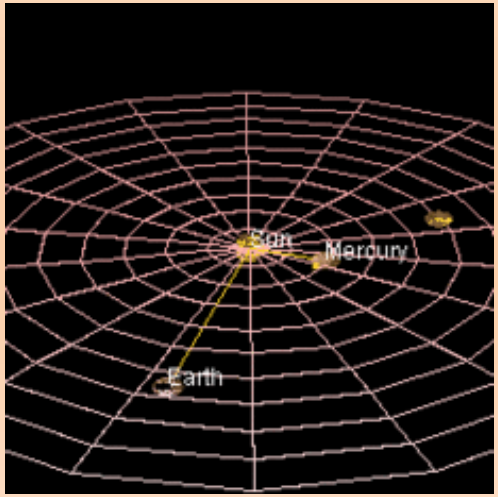
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Experimental Verification

The bending light

Since we postulate that gravity is a curvature of spacetime and that photons follow geodesics in spacetime, we find that “gravity bends light” (to be precise, it is the spacetime which is bent). The effect was experimentally verified in the famous 1919 eclipse expedition of Arthur Eddington. During a total eclipse of the Sun, the position of a star very near the Sun’s limb was measured. The star’s position was found to be shifted by an amount predicted by the general theory.





Mercury orbits

Mercury deviates from the precession predicted from these Newtonian effects: the rate of the precession disagreed from that predicted from Newton's theory by 43" (arcseconds= 1/3600 of a degree) per tropical century. In general relativity, this remaining precession, or change of orientation of the orbital ellipse within its orbital plane, is explained by gravitation being mediated by the curvature of spacetime.

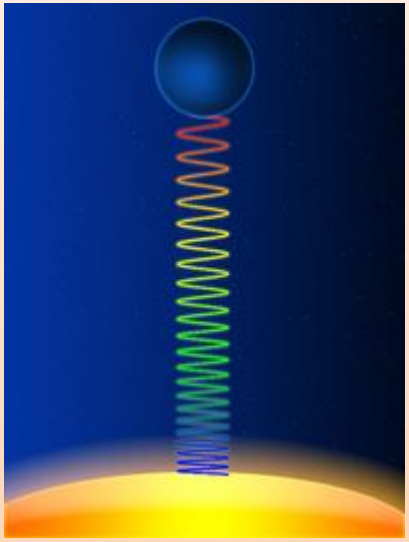


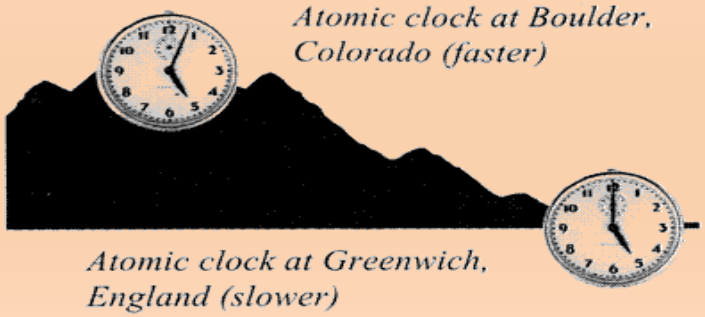
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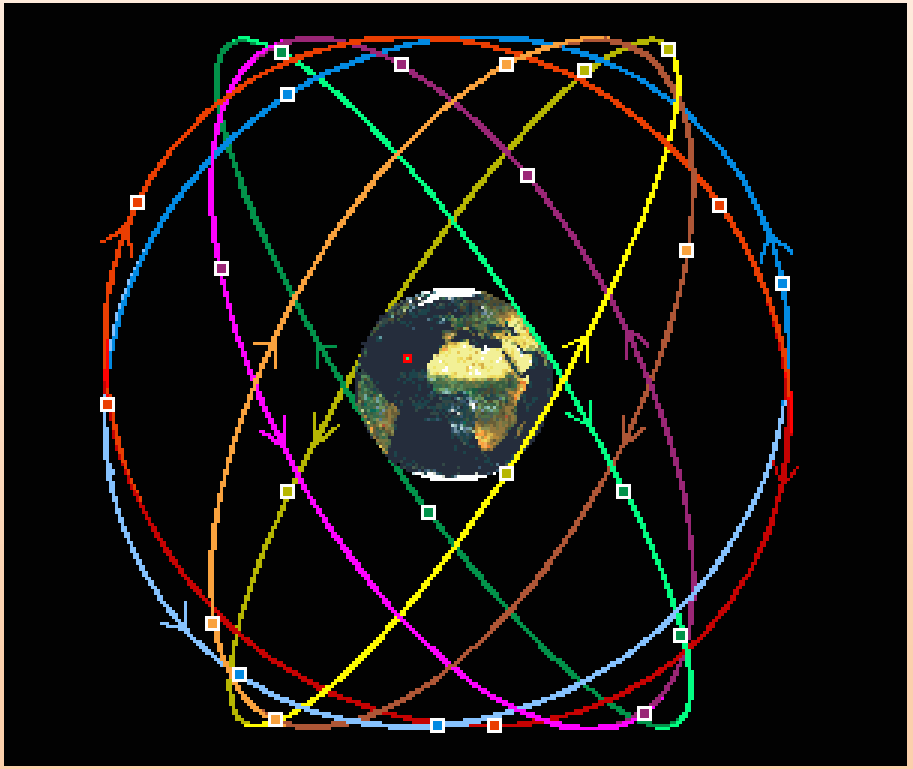


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Experimental Verification

GPS

The Global Positioning System (gps) are satellites that form Launched by the US Air Force in orbit around the Earth, they constantly send radio signals down to Earth that can be used in navigation: with a gps receiver one can pinpoint one's location to within 10m, an extraordinary accuracy. The satellites carry precise atomic clocks, the most accurate clocks that can be made. Because of the effect of gravity on time, these tick faster than do clocks on the ground; the difference is about three microseconds per day. Therefore, this redshift correction must be taken into account in order for the system to function.





R. Pancari

Learning Check 8

A. Gravity is a curvature in space-time true/false

B. Tensors are associated to mass true/false

C. Force produce gravitational waves true/false

D. Black holes are generated by small stars true/false

E. The gravitational red shift slows clocks true/false

F. The GPS consider a redshift correction true/false



R. Pancani

Solution 8

A. Gravity is a curvature in space-time **true**/false

B. Tensors are associated to mass true/**false**

C. Force produce gravitational waves true/**false**

D. Black holes are generated by small stars true/**false**

E. The gravitational red shift slows clocks **true**/false

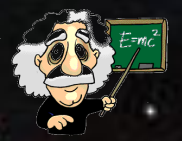
F. The GPS consider a redshift correction **true**/false



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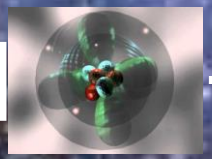
Quantum Theory



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Background image from http://www.huffingtonpost.com/2014/11/04/parallel-universes-quantum-mechanics-theory_n_6091438.html

Quantum Theory

- Origins
  - 
  - Continuous – discrete
  - Unsolved problems
  - Atomic models
- Quantum Mechanic
  - 
  - Wave–particle duality
  - Uncertainty principle
  - Schrödinger Equation

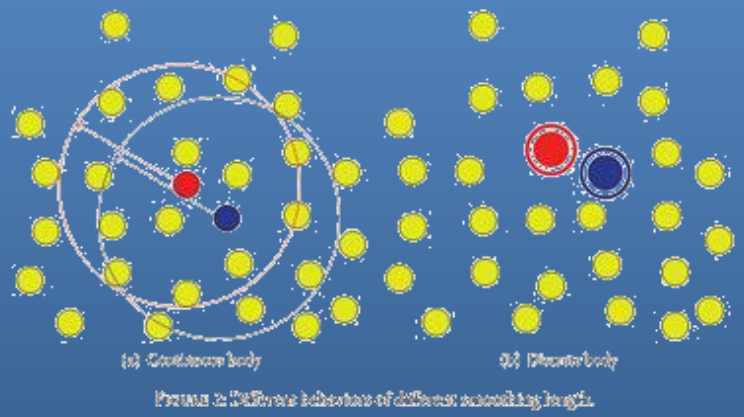
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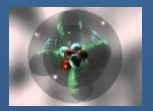
Continuous Vs discrete

- A continuous magnitude can not be expressed by an integer, but only by a real number. It is infinitely divisible, for example is a continuous magnitude the distance between two points, in fact between two points taken indefinitely neighbors, there are always infinite points and you can not define a "minimum level" under which this law lose meaning.

A discrete magnitude, however, can be experienced by a whole natural positive or negative number, it can vary only by leaps, such as the number of inhabitants of a city.

A quantity can be studied as a discrete body, as it consists of elementary particles distinct from each other, or as a continuous body, since the large number, the cohesion and the interdependence between these particles make disappear any granularity, at least at the macroscopic level. In macrophysics we observe continuous magnitude but the reality is not a continuous system: the molecular structure must be represented as a finite system, whose molecules are enumerable, occupy a certain region of space.

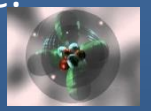




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Continuous Vs discrete

Classical physics refused discontinuity, providing causality, according to the Galileo's axiom *"natura non facit saltus"* approved also by Libniz, who contrasted causality by introducing the randomness through probability and statistics.
According to the classical physics some quantities such as the emission and absorption of radiation by matter belonged to the group of those continuous, while, according to the new theories of Plank these quantities are characterized by jumps to certain values, the multiples of the elementary quantum of energy; then you can say that with the new theories it has gone from a world interpreted only continuously to a well played discreetly. Even a physical system such as the electromagnetic field, which is apparently a continuous system, is a discrete system of photons, calculated knowing their average number for each physical state. You could say that the only truly continuous system is the space-time, but Albert Einstein showed, that the concept of space and time are not absolute but relative, meaning they depend on the system reference in which the observer is located, and constitute the space-time continuum , but it can be analyzed with the Planck scale, which examines the spatio-temporal distances in terms of very small and one-dimensional things, which represent the "granularity" of space-time.



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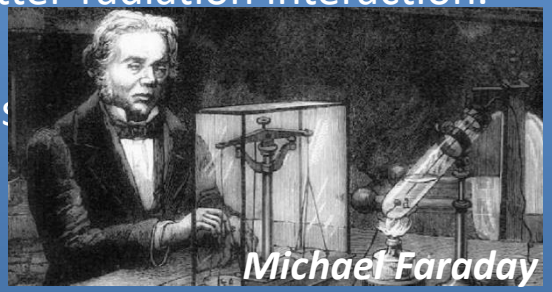
The first elementary quantum of electricity

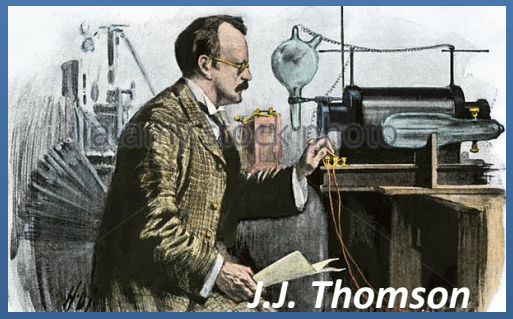
The word *quantum* derives from the Latin, meaning "how great" or "how much"
Quantum theory, together with Relativity, is the theoretical basis of modern physics that explains the nature and behaviour of matter and energy on the atomic and subatomic level,

Short history of electron (the first elementary quantum of electricity)

The experimental efforts at the end of '800 are aimed at the study of the structure of matter, the mutual interactions between bodies and the matter-radiation interaction.

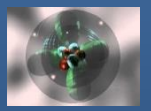
- The discrete nature of electric charge was proposed by ***Michael Faraday***, in 1830, his electrolysis experiments
- ***Goldstein***, in 1886, discovered the canal rays in residual gas ions


Michael Faraday


J.J. Thomson

- ***J.J. Thomson***, in 1897 discovered the electrons cathode rays
- ***Robert Millikan*** in 1910, showed that the negative electron charge ***e*** was quantified (***e*** = $1{,}602 \cdot 10^{-19}$C)

Following these discoveries the existence of atoms and molecules as a working hypothesis becomes reality.

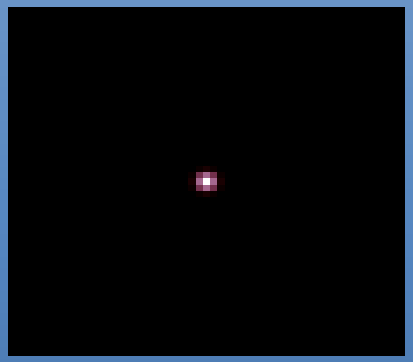


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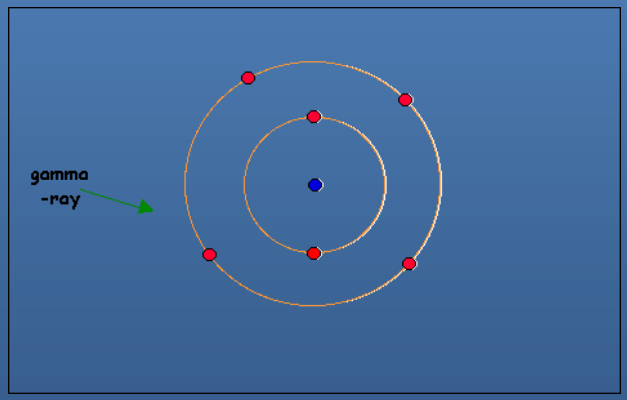
Classical unsolved problems

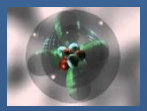
Microscopic knowledge becomes more and more precise and appear difficulties and contradictions of the classical theory, that failed to explain problems as the photoelectric effect and blackbody spectrum

Blackbody radiation or cavity radiation refers to a system which absorbs all radiation incident upon it and re-radiates energy which is characteristic of this radiating system only, not dependent upon the type of radiation which is incident upon it. The radiated energy can be considered to be produced by standing wave or resonant modes of the cavity which is radiating.



The ***Photoelectric effect*** refers to the emission, or ejection, of electrons from the surface of, generally, a metal in response to incident light. Energy contained within the incident light is absorbed by electrons within the metal, giving the electrons sufficient energy to be 'knocked' out of, that is, emitted from, the surface of the metal.





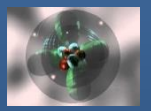
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Blackbody radiation

One experimental phenomenon that could not be adequately explained by classical physics was black-body radiation. A black body emits radiation at all frequencies, but not all at the same rate, and the exact spectral radiance of the object is determined by its temperature. The amount of radiation emitted in a given frequency range should be proportional to the number of modes in that range. The best of classical physics suggested that all modes had an equal chance of being produced, and that the number of modes went up proportional to the square of the frequency.

But the predicted continual increase in radiated energy with frequency, dubbed the "ultraviolet catastrophe“, did not happen.

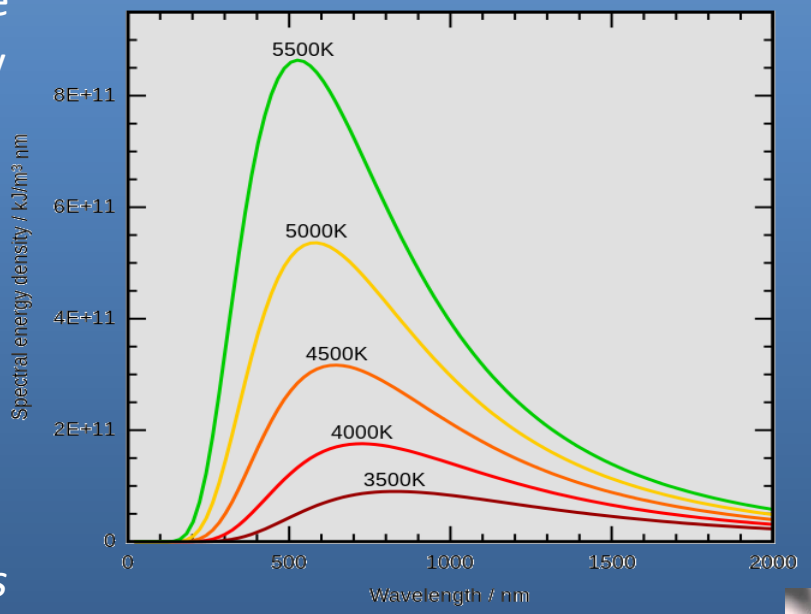




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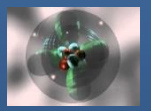
Wien's law

Wien's law states that the black body radiation curve for different temperatures peaks at a wavelength inversely proportional to the temperature. The shift of that peak is a direct consequence of the Planck radiation law which describes the spectral brightness of black body radiation as a function of wavelength at any given temperature. However it had been discovered by Wilhelm Wien in 1894 several years before Max Planck developed that more general equation, and describes the entire shift of the spectrum of black body radiation toward shorter wavelengths as temperature increases.Formally, Wien's displacement law states that the spectral radiance of black body radiation per unit wavelength, peaks at the wavelength λ_{max} given by:



$$\lambda_0 T = \text{constant}$$

(T= body absolute temperature, *Wien's displacement constant* is equal to 2.9×10^{-3} m·K)

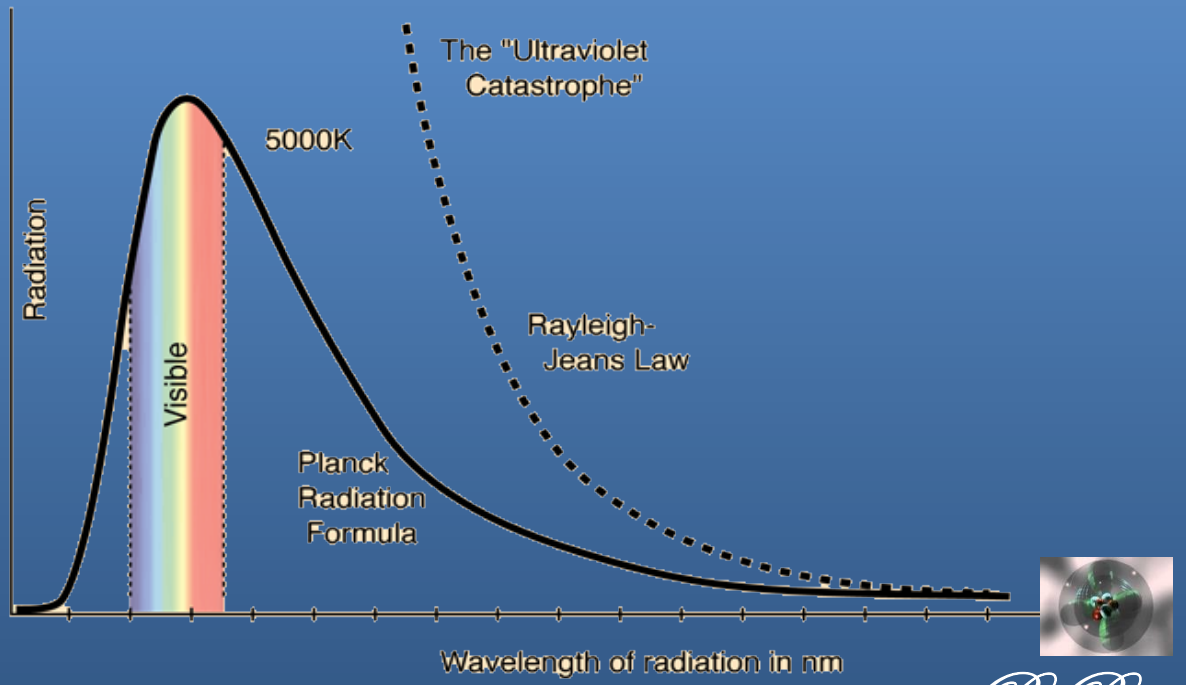


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Ultraviolet catastrophe

Lord Rayleigh and J. H. Jeans developed an equation which explained blackbody radiation at low frequencies. The equation which seemed to express blackbody radiation was built upon all the known assumptions of physics at the time. **The big assumption which Rayleigh and Jean implied was that infinitesimal amounts of energy were continuously added to the system when the frequency was increased.** Classical physics assumed that energy emitted by atomic oscillations could have any continuous value.

The experimental results showed a bell type of curve but according to the *Rayleigh-Jeans law* the frequency diverged as it neared the ultraviolet region. This inconsistency was termed the *ultraviolet catastrophe*



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Planck Radiation Formula

Planck modelled the walls of the cavity as a collection of electric oscillators. He assumed that all frequencies would be present. He also expected that the average frequency increases with increasing temperature since heating the walls causes the oscillators to vibrate faster and faster until thermal equilibrium is reached. The electromagnetic theory could tell everything about the emission, absorption and propagation of the radiation but nothing about the energy distribution at equilibrium. Planck first derived an empirical formula to bridge the gap between Wien's law and Rayleigh Jeans´ law of black-body radiation and then searched for a rigorous derivation of his formula.

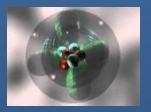
Energy per unit volume per unit frequency S_f:

$$S_f = \frac{8\pi h}{c^3} \frac{f^3}{e^{\frac{hf}{kT}} - 1}$$

Planck postulated that, for the energy ***E*** of an oscillator of frequency ***f,*** the variation of energy had to be quantized:

$$\Delta E = hf \quad (h=6.626 \times 10^{-34}\ Js)$$

He found that blackbody radiation was quantized.

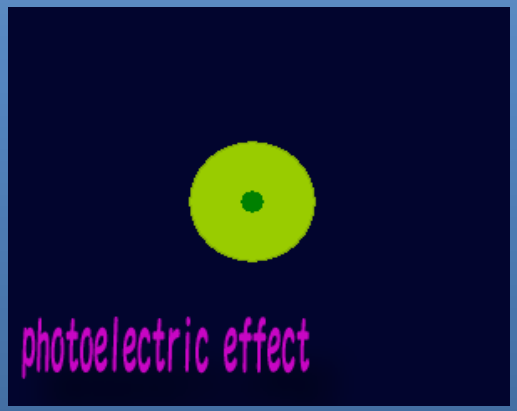


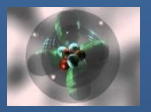
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Photoelectric effect

In 1880 Hertz and Lenard observed that when a clean metallic surface is irradiated by monochromatic light of proper frequency, electrons are emitted from it. This phenomenon was called as Photoelectric Effect. Using the classical Maxwell wave theory of light, there were some **QUESTIONS**:

1. The electrons were emitted immediately - no time lag!
2. Increasing the intensity of the light increased the number of photoelectrons, but not their maximum kinetic energy!
3. Red light will not cause the ejection of electrons, no matter what the intensity!
4. A weak violet light will eject only a few electrons, but their maximum kinetic energies are greater than those for intense light of longer wavelengths!

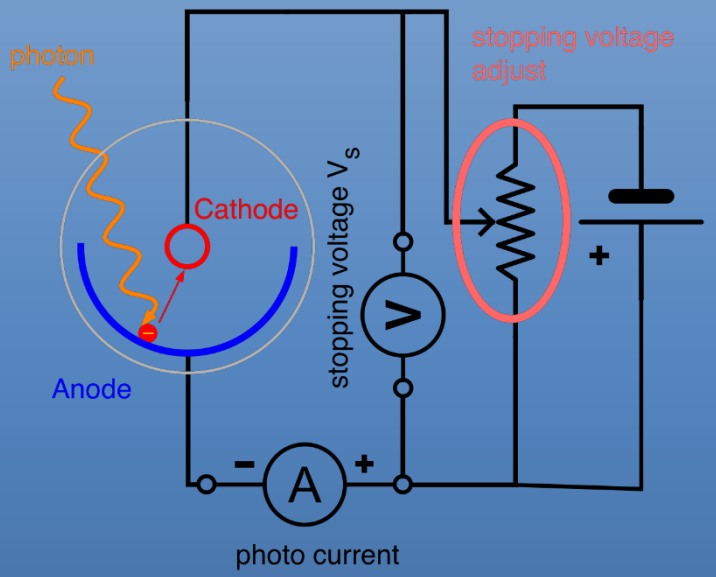


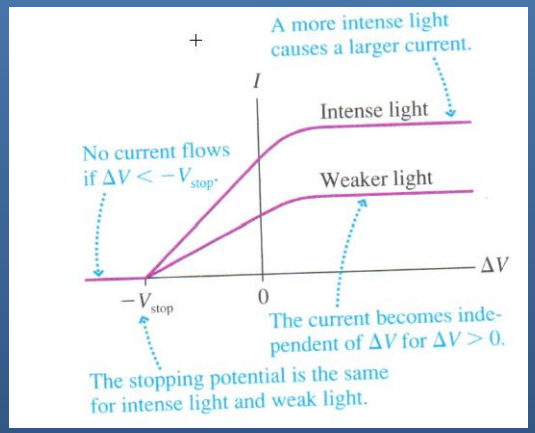


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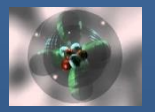
The experiment

The experiment demonstrates the photoelectric effect, for which Einstein was awarded a Nobel prize, and in the process determines Planck's constant.
A photon at a high frequency hits the conducting anode and knocks out an electron. All electrons that have sufficient kinetic energy to reach the cathode produce an electric current. The adjustable stopping voltage determines the minimal kinetic energy the electrons need equal to ***W*** , the work function. current.





The goal of the analysis is to carefully determine the minimal retarding voltage from which one can determine kinetic energy of the ejected photoelectrons, that, intercepted by the collector, generate a measurable.



R. Pancari

Photoelectric effect

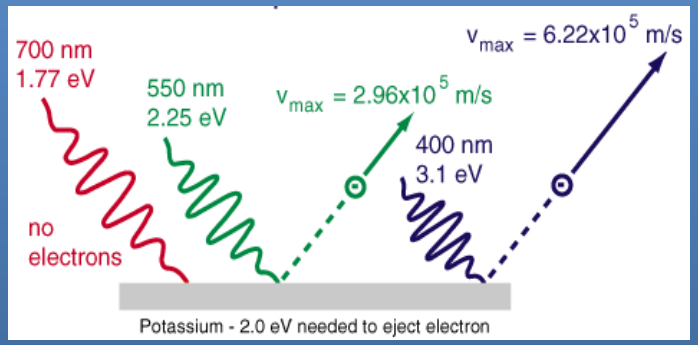
Einstein in 1905 resolved the questions by proposing that the incident light consisted of individual ***quanta***, called ***photons,*** that interacted with the electrons in the metal like discrete particles, rather than as continuous waves. For a given frequency, or 'color,' of the incident radiation, each photon carried the energy:

$$E = hf$$ (h = Planck's constant ; f = frequency)

Einstein equation adds to the photon energy the threshold energy ***W*** :

$$E + W = hf$$

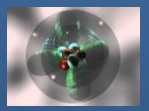
(W = hv_0, work function, minimum energy required to remove an electron from the surface of a metal; E = maximum kinetic energy of ejected electrons)
No electron will be emitted if the incident photon's energy is less than the work function.



Applying Einstein's special theory of relativity, the relation between energy (***E***) and momentum (***p***) of a particle of rest mass (m_0) is:

$$E^2 = (pc)^2 + (m_0c^2)^2$$

so the $p = \frac{hf}{c}$

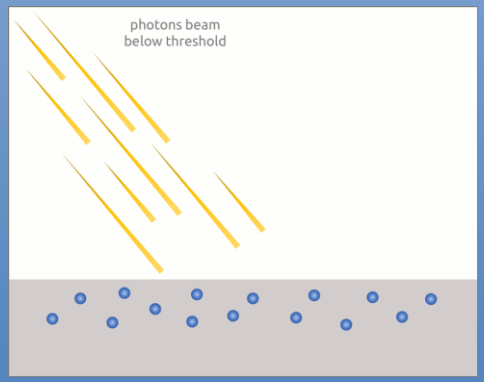


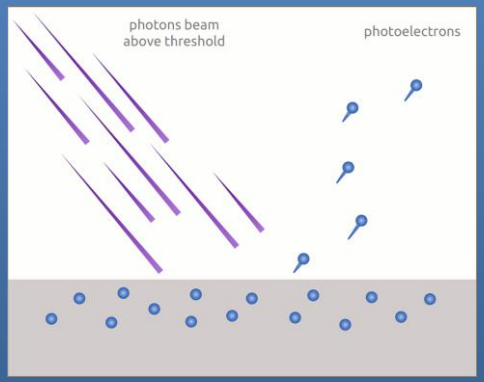
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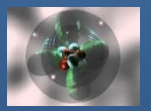
Photoelectric effect

ANSWERS:

1. The time between the incidence and emission of a photoelectron is very small, less than 10^{-9} second.
2. The rate at which photoelectrons are ejected is directly proportional to the intensity of the incident light, for a given frequency of incident radiation and metal.
3. For a given metal, there is a minimum frequency of incident radiation below which the effect will not occur so no photoelectrons can be emitted (threshold frequency).
4. Above the threshold frequency, the maximum kinetic energy of the emitted photoelectron depends on the frequency of the incident radiation but is independent of its intensity.







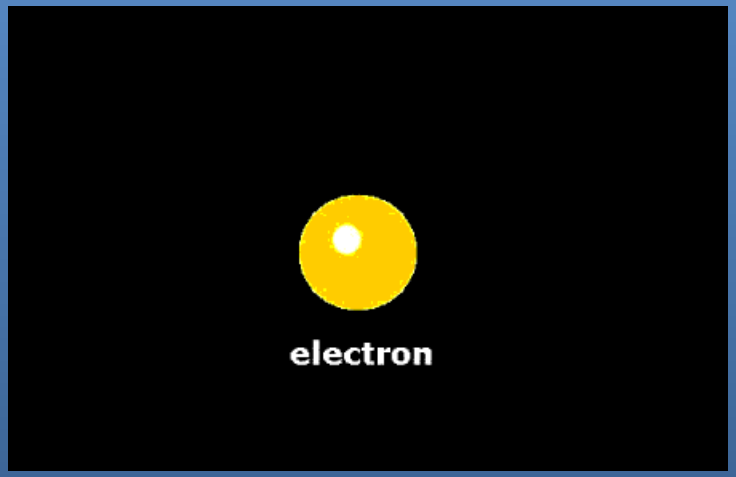
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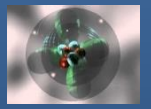
Compton effect

The American physicist Arthur Holly Compton explained (1922; published 1923) the wavelength increase by considering X-rays as composed of discrete pulses, or quanta, of electromagnetic energy. The American chemist Gilbert Lewis later coined the term *photon* for light quanta. In the Compton effect, individual photons collide with single electrons that are free or quite loosely bound in the atoms of matter.

Colliding photons transfer some of their energy and momentum to the electrons, which in turn recoil. In the instant of the collision, new photons of less energy and momentum are produced that scatter at angles the size of which depends on the amount of energy lost to the recoiling electrons.

Because of the relation between energy and wavelength, the scattered photons have a longer wavelength that also depends on the size of the angle through which the X-rays were diverted. The increase in wavelength, or Compton shift, does not depend on the wavelength of the incident photon.

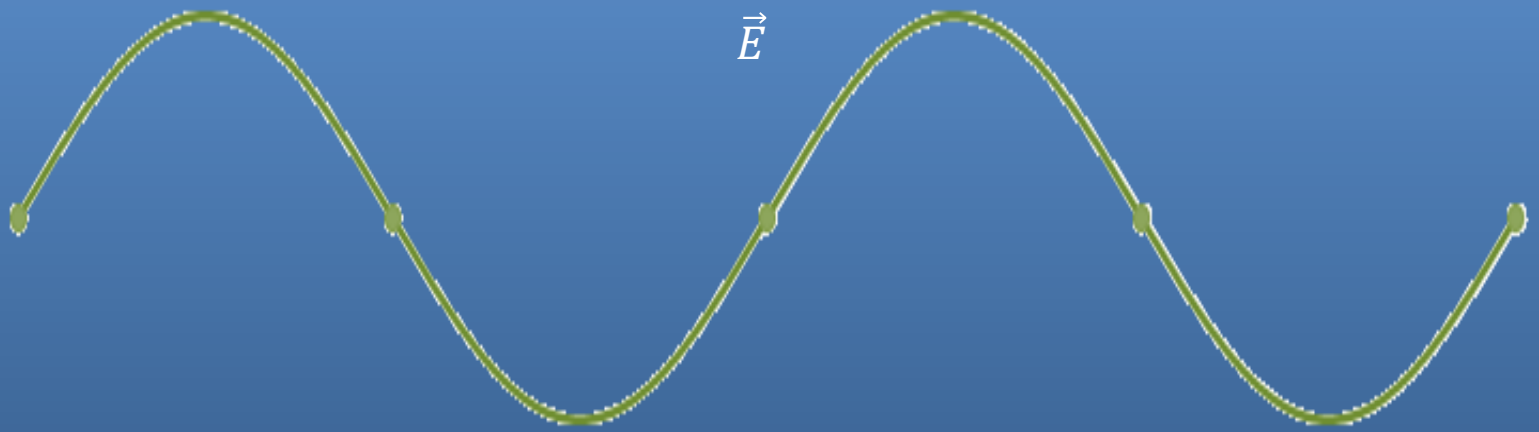




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Learning Check 1

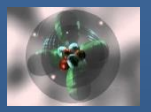
What is the energy, in eV(1 eV = 1.60 x 10^{-19} J), of a photon of yellow-green light, λ= 555 nm?



Solution 1

Photon energy

Given	Requests	Work
λ= 555 nm **Variables** λ **Constants** c = 3x10^{8}m/s h = 6.626x10^{-34}Js 1 eV = 1.60 x 10^{-19} J	**E(eV) = ??**	**First we find *f* from wave equation: $c = f\lambda$** **After we use the formulas:** $f = \frac{c}{\lambda}; \quad E = hf = \frac{hc}{\lambda}$ $E = \frac{(6.626 \times 10^{-34}\,\mathrm{J\cdot s})(3 \times 10^{8}\,\mathrm{m/s})}{555 \times 10^{-9}\ \mathrm{m}}$ *E = 3.58 x 10^{-19} J* → *E = 2.24 eV*

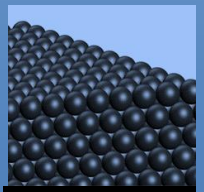


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Atomic structure

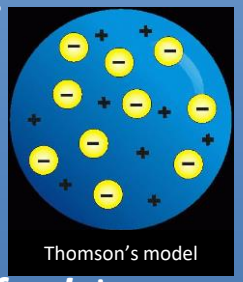
Short history of atom:

Around the year 430 b.C., a Greek philosopher named ***Democritus*** proposed the idea that matter is formed of small pieces that could not be cut into smaller parts. He used the word **atomos**, which means "uncuttable," for these smallest possible pieces. The Greek idea of atoms had to wait about 2,000 years before it became accepted.



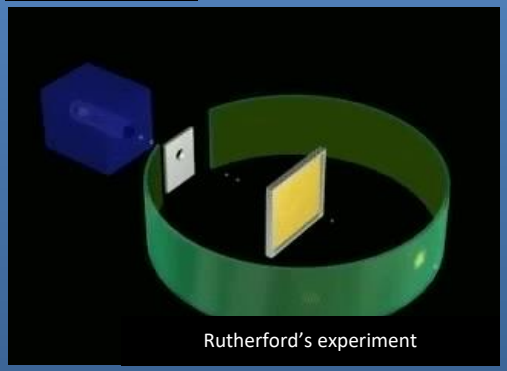
Dalton's model

In **1808** ***Dalton*** thought that atoms were like smooth, hard balls that could not be broken down into smaller pieces.



Thomson's model

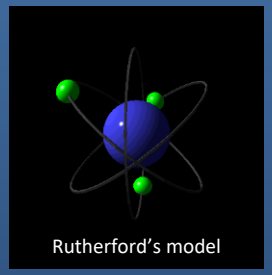
In **1897** ***J. J. Thomson*** suggested that atoms had negatively charged embedded in a positive sphere, the *"plum pudding"* model.



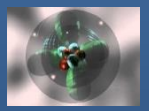
Rutherford's experiment

In **1911,** one of Thomson's students, ***Ernest Rutherford***, in an experiment bombarded thin gold foils with heavy alpha positive particles most went through, some were ben,t a few bounced back, the results inferred that an atom's positive charge must be clustered in a tiny region in its center, called the **nucleus**.

In this planetary model the atom was mostly empty space with electrons moving around the nucleus in that space. Later, he named the positively charges in the nucleus of an atom **protons.**



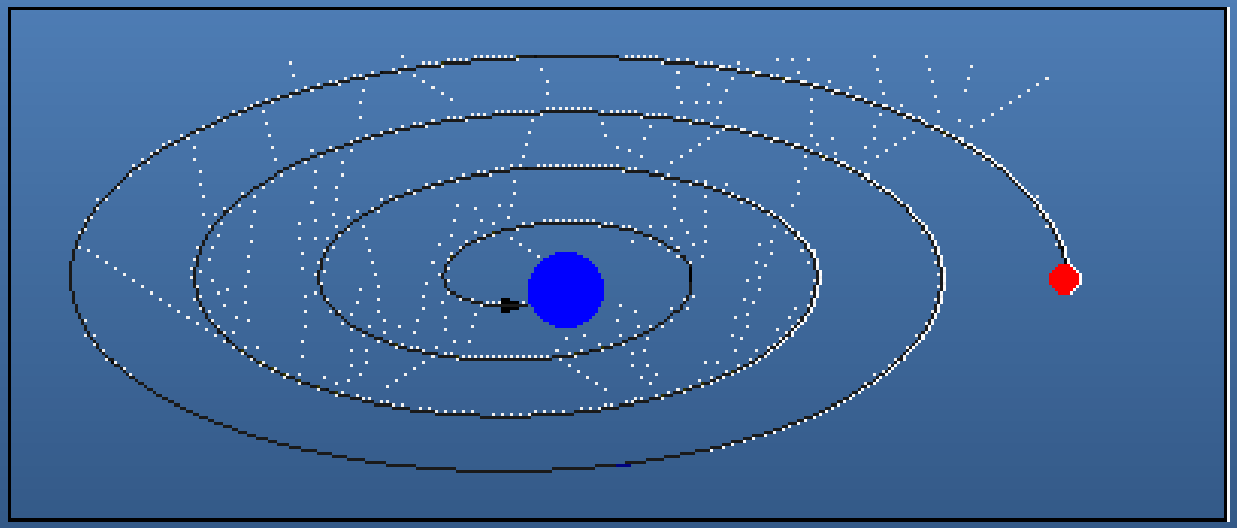
Rutherford's model



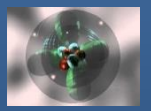
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Limits of Rutherford's model

The atomic model proposed by Rutherford suffered from an electromagnetic instability and mechanical instability: since the electron in its motion around the positive nucleus is subject to acceleration, it radiates electromagnetic energy of the same frequency of its Revolution motion, losing energy and then falling on the nucleus with a spiral motion. In the case of heavier atoms, around which more electrons rotate, the latter would have been subjected to electrostatic repulsion which would also render their orbits mechanically unstable and any external perturbation would have been sufficient to alter the distribution of electrons in atoms. It was Niels Bohr to solve the difficulties of the Rutherford model



The electron should fall on the nucleus.

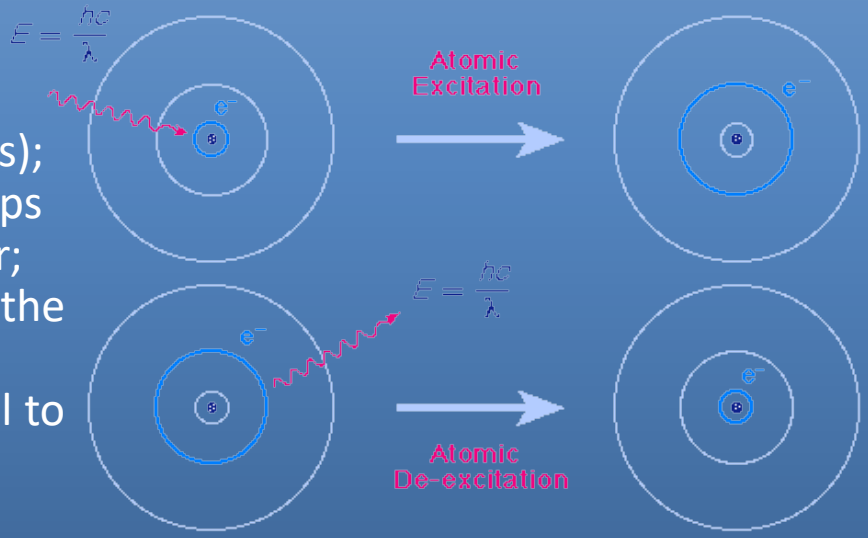


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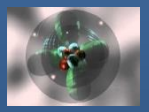
Bhor-Sommerfeld's atomic model

In **1913**, the Danish scientist ***Niels Bohr,*** a student of both Thomson and Rutherford, revised the atomic model. Observing hydrogen emission and absorption spectra, Bohr proposes an atomic model (planetary like Rutherford's, but with ***quantized orbits*** according to Plank, valid only for the hydrogen atom), which is based on the following points:

1. The electron passes only certain quantized circular orbits, in which rotates without absorbing or emitting energy (stationary orbits);
2. The electron absorbs energy only when it jumps from one orbit to another energy level greater;
3. If the electron returns to a lower energy level the atom emits energy below photon shape;
4. The light energy, emitted or absorbed, is equal to the energy difference of the two orbits;
5. Each jump is revealed by a row of spectrum



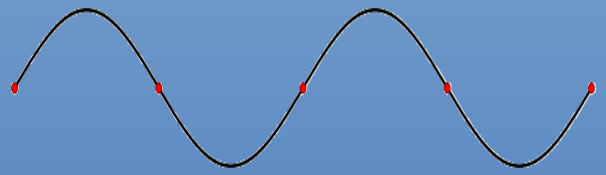
In **1915** the German physicist ***A.J. Sommerfeld*** hypothesized that the electron moved by describing elliptical orbits in which the nucleus occupied one of the two fires. After the concept of orbit leaves the concept of orbitals. Some problems existed with Bohr-Sommerfeld theory: it only successful for the H atom and it introduced quantum idea artificially.

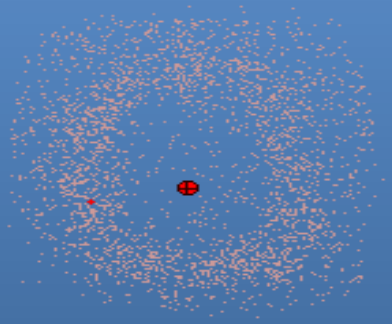


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Quantum atomic model

The problems were solved by quantum or wave mechanics.
The ***wave model***: quantum mechanics shows that it is not possible to define the trajectory of an electron, which has a delocalized movement of a wave-particle.
According to **Louis de Broglie** theory all particles could be treated as matter waves with a wavelength given by the following equation: **$\lambda = h/mv$**





In the quantum model the space in which electrons are likely to be found is called ***electron cloud***. Electrons whirl about the nucleus billions of times in one second.
Location of electrons depends upon how much energy the electron has.

In ***1926 Erwin Schrödinger***, an Austrian physicist, used a mathematical equations to describe the likelihood of finding an electron in a certain position. This atomic model is known as the ***quantum mechanical model*** of the atom. In the same year ***Max Born*** defines the atomic orbital as a region around the nucleus where the electron can be found at a given instant, with a very high probability (95%). In **1932**, **James Chadwick** discovered the ***neutron***.



R. Pancari

Learning Check 2

A. Who did propose the idea of the atoms?

1) Democritus 2) Dalton

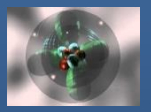
B. Which of atomic structure is called planetary?

1) Rutherford 2) Bohr

C. In the electron cloud model:

1) Electrons rotate some seconds in one second

2) Electrons location depends on their energy



R. Pancari

Solution2

A. Who did propose the idea of the atoms?

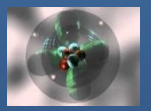
1) Democritus

B. Which of atomic structure is called planetary?

1) Rutherford

C. In the electron cloud model:

2) Electrons location depends on their energy



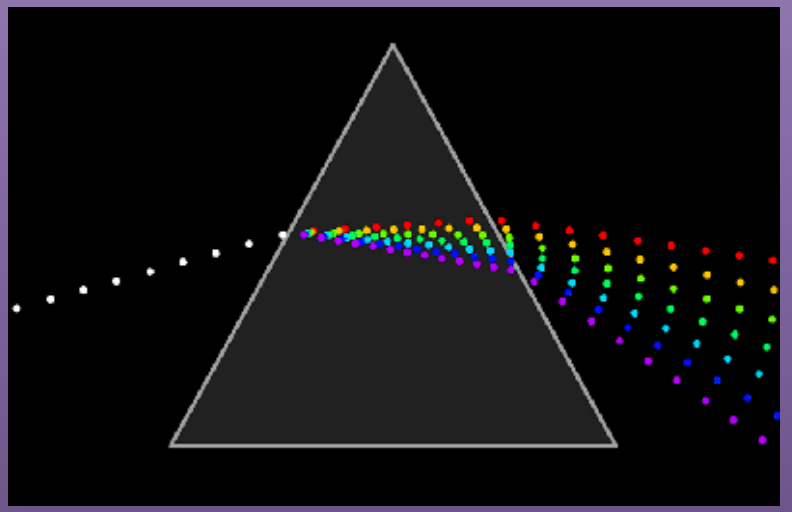
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Nature of light

Newton's corpuscular theory:

It was proposed by ***Isaac Newton*** in ***1675***, justifying reflection and refraction, but not the diffraction that he attributed to *inflexion* of light rays. His theory is based on the following points:

1. Light consists of very tiny, elastic particles known as "corpuscles".
2. These corpuscles are emitted from a source of light in all directions and travel in straight line with high constant velocity in the same medium.
3. When these particles enter the eyes, they produce image of the object or sensation of vision.
4. Corpuscles of different colours have different sizes.



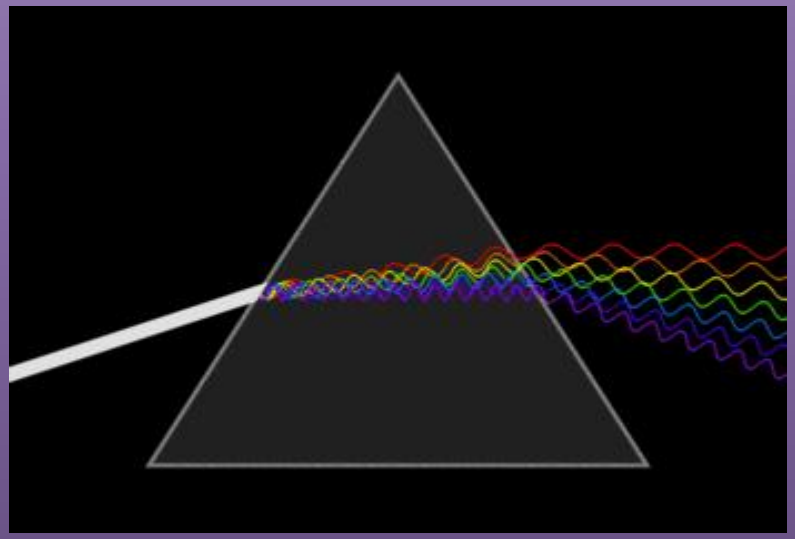


R. Pancari

Nature of light

***Huygen's wave theory*:**
In ***1679, Christian Huygens*** proposed the wave theory of light, the main features of it are as follows:

1. A medium is necessary for the propagation of waves & the whole space is filled with an imaginary medium called Ether.
2. Each point in a source of light sends out waves in all directions in the hypothetical medium Ether.
3. Light is a form of energy.
4. Light travels in the form of waves.
5. Light waves have very short wave length.





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X-ray diffraction

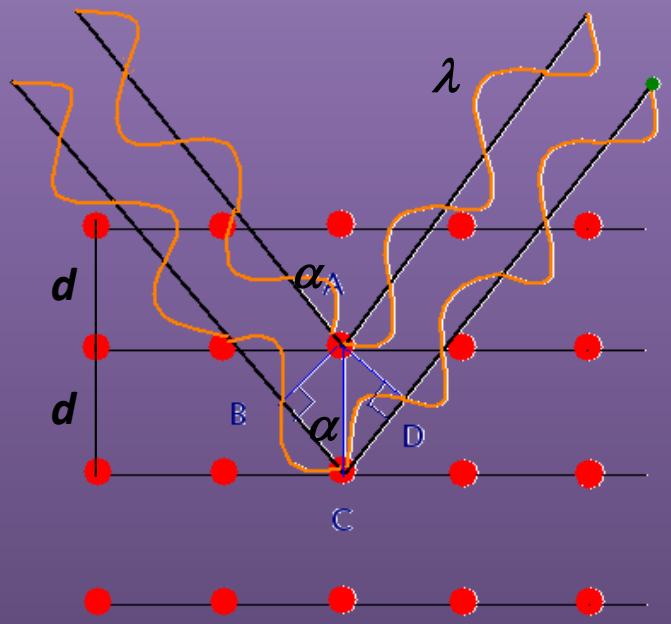
In **1895**, a German physicist, **W. C. Roentgen** discovered X-ray that would pass through the tissue of humans leaving the bones and metals visible, but their wavelength is short comprised between 10^{-12}m and 10^{-8}m so it was hard to prove their ***wave-like nature***.

In **1912 von Laue** proved the diffraction of X-ray, using crystals that have typical interatomic spacing ~ 2-3 Å (Å = 10^{-10} m).

The analysis of diffraction produced figures is complex but is based on a simple law formulated by **W. L. Bragg**, for a **constructive interference** is:

$$2d\,sen\alpha = n\lambda \qquad n = 1, 2, 3, \ldots$$

(d = distance between reticular planes; α = angle of incidence; λ = wavelength)





R. Pancari

Nature of light

Quantum theory:

It was put forward in ***1905*** by ***Max Planck.***

According to this theory:

1. Energy radiated or absorbed can not have any fractional value. This energy must be an integral multiple of a fixed quantity of energy, called ***quantum***.
2. Energy released or absorbed is always in the form of packets of energy or bundles of energy. These packets of energy are known as photons or ***quanta***



packets of energy photons or quanta

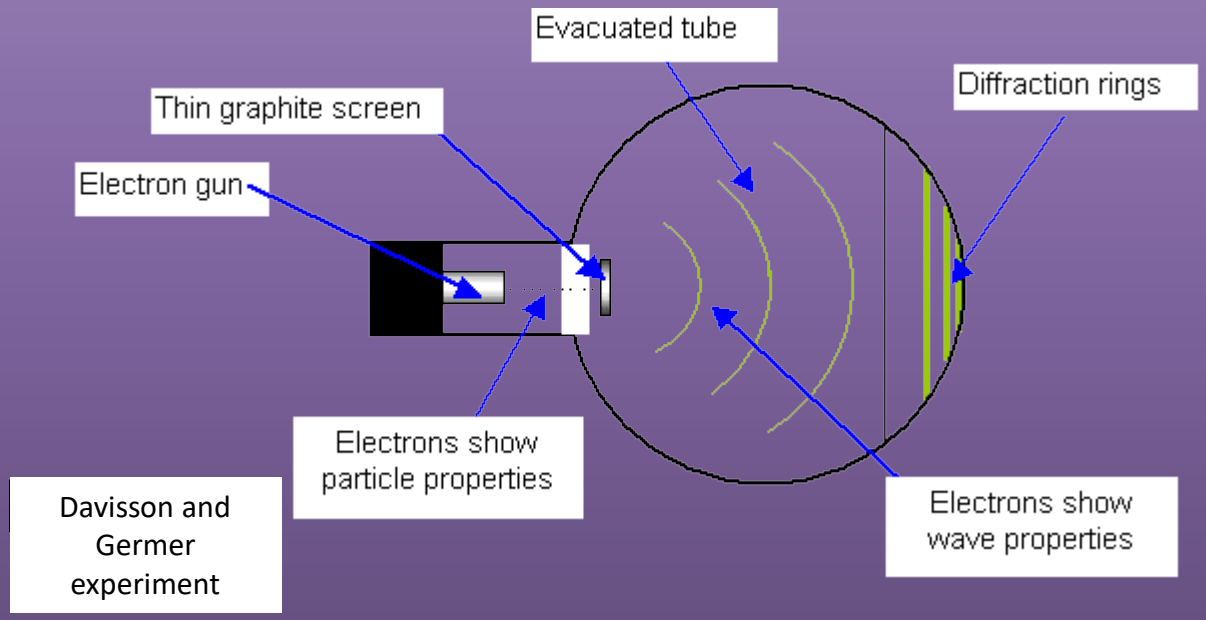
Louis de Broglie in ***1923*** proposed that matter particles should exhibit wave properties just as light waves exhibited particle properties. These waves have very small wavelengths in most situations so that their presence was difficult to observeHe postulated that a particle with mass **m a**nd a velocity **v** has an associated wavelength. The equation $\lambda = h/mv$ is called the **de Broglie relation**.



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Electron diffraction

In **1927**, **Davisson** and Germer demonstrated that a beam of electrons, just like X rays, could be diffracted by a crystal, producing diffraction rings.





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Wave-particle duality

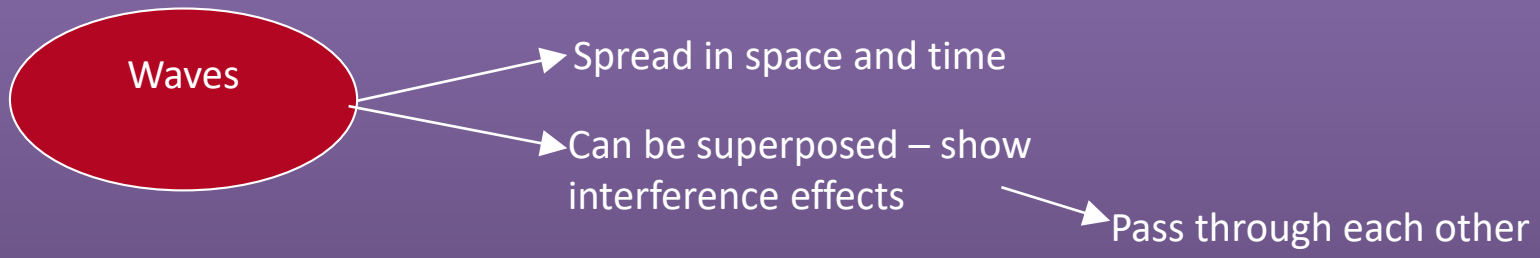
The behavior of a "microscopic" particle is very different from that of a classical particle:

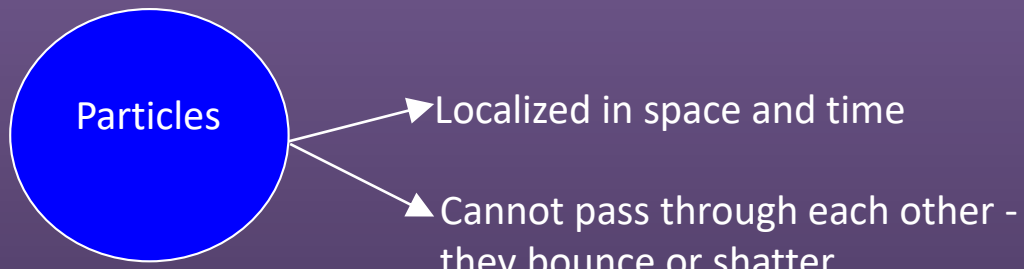
➔ in some experiments it resembles the behavior of a classical wave (not localized in space)

➔ in other experiments it behaves as a classical particle (localized in space)

Corpuscular theories of light treat light as though it were composed of particles, but can not explain ***diffraction*** and ***interference***.

Maxwell's theory of electromagnetic radiation can explain these two phenomena, which was the reason why the corpuscular theory of light was abandoned.







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Summery of nature duality

Wave nature

LIGHT

Particle **nature**

Optical microscope

Interference

Convert light to electric current

Photo-electric effect

PARTICLES

Microscope

Matter waves -electron

Discrete (Quantum) states of confined systems, such as atoms.

Electric current

photon-electron collisions



R. Pancari

Learning Check 3

A. The light has only a wave-like nature true/false

B. The photons are also called quanta true/false

C. In Bragg's law the angle α depends on λ true/false

D. The electron has only a particle-like nature true/false

E. The particles pass through each other true/false

F. The waves show interference effects true/false



R. Pancari

Solution 3

A. The light has only a wave-like nature true/false

B. The photons are also called quanta true/false

C. In Bragg's law the angle α depends on λ true/false

D. The electron has only a particle-like nature true/false

E. The particles pass through each other true/false

F. The waves show interference effects true/false



R. Pancari

Heisenberg's uncertainty principle

In **1927 W. Heisenberg** formulated the following principle ***"locating a particle in a small region of space makes the momentum of the particle uncertain; and conversely, that measuring the momentum of a particle precisely makes the position uncertain".*** So we can say that the product of the uncertainty in position Δx and the uncertainty in momentum $\Delta p = m\Delta v_x$ of a particle can be no smaller than a constant equal to $\mathbf{h/4\pi}$:

$$(\Delta x)(m\Delta v_x) \geq \frac{h}{4\pi}$$

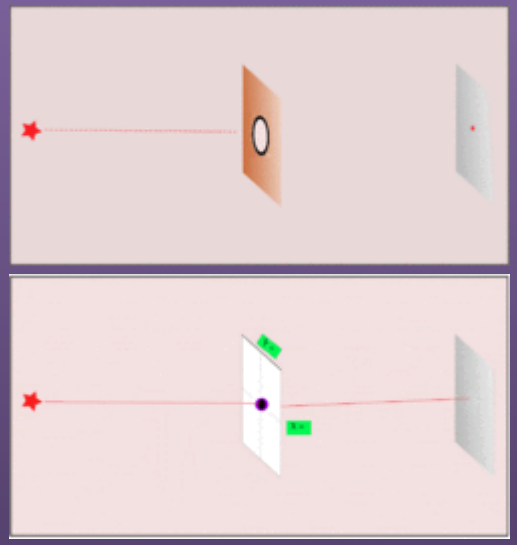
The principle can be apply to the uncertainty in time Δt and in energy ΔE :

$$(\Delta t)(\Delta E) \geq \frac{h}{4\pi}$$

When the mass is large, for example a baseball in macrophysics, the uncertainties are small, but for electrons or photons in microphysics high uncertainties disallow defining an exact orbit.

Narrowing the gap increases the certainty of where the photon is in the middle, but then its direction from there to the detection screen on the right becomes correspondingly more uncertain.

Suspending the center gap by springs lets momentum be measured, but unpredictably moves the gap so information on the photon's location in the middle gets lost.





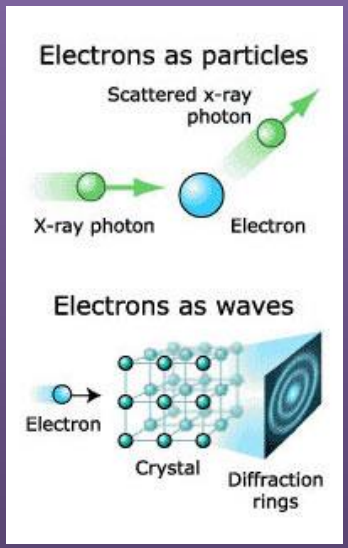
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Complementarity principle

A complete knowledge of phenomena on atomic dimensions requires a description of both wave and particle properties. Depending on the experimental arrangement, the behaviour of such phenomena as light and electrons is sometimes wavelike and sometimes particle-like; such things have a wave-particle duality.

In ***1928*** the Danish physicist ***Niels Bohr*** announced the principle: ***on the atomic level a physical phenomenon expresses itself differently depending on the experimental setup used to observe it.*** It is impossible to observe both the wave and particle aspects simultaneously. Together, however, they present a fuller description than either of the two taken alone.

In effect, the complementarity principle implies that phenomena on the atomic and subatomic scale are not strictly like large-scale particles or waves (*e.g.*, billiard balls and water waves). Such particle and wave characteristics in the same large-scale phenomenon are incompatible rather than complementary. Knowledge of a small-scale phenomenon, however, is essentially incomplete until both aspects are known.



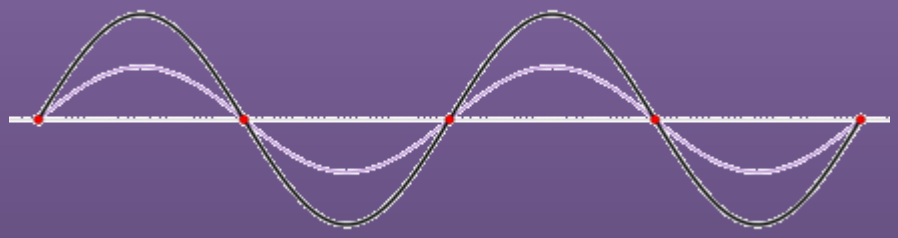


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Superposition

The fact that quantum systems, such as electrons and protons, have indeterminate aspects means they exist as possibilities rather than actualities. This gives them the property of being things that might be or might happen, rather than things that are. This is in sharp contrast to Newtonian physics where things are or are not, there is no uncertainty except those imposed by poor data or limitations of the data gathering equipment. Further experimentation showed that reality at the quantum (microscopic) level consists of two kinds of reality, actual and potential. The actual is what we get when we see or measure a quantum entity, the potential is the state in which the object existed before it was measured. The result is that a quantum entity (a photon, electron, neutron, etc) exists in multiple possibilities of realities known as superpositions.

The superposition of possible positions for an electron can be demonstrated by the observed phenomenon called quantum tunneling.



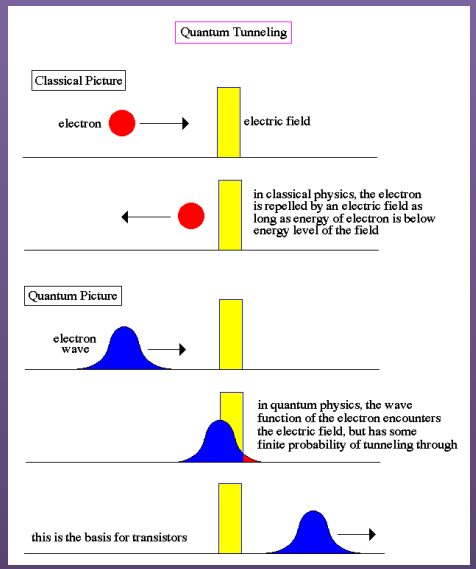


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Quantum tunneling

This phenomenon is an important consequence of quantum mechanics. Consider a particle with energy E in the inner region of a one-dimensional potential well V(x) (a potential well is a potential that has a lower value in a certain region of space than in the neighboring regions). In classical mechanics, if E < V (the maximum height of the potential barrier), the particle remains in the well forever; if E > V , the particle escapes. In quantum mechanic the particle can escape even if its energy E is below the height of the barrier V , although the probability of escape is small unless E is close to V, in that case, the particle may tunnel through the potential barrier and emerge with the same energy E.

The phenomenon has many important applications. For example, it describes a type of radioactive decay in which a nucleus emits an alpha particle, the alpha particle is confined before the decay by a potential. For a given nuclear species, it is possible to measure the energy E of the emitted alpha particle and the average lifetime of the nucleus before decay. The lifetime of the nucleus is a measure of the probability of tunneling through the barrier--the shorter the lifetime, the higher the probability.





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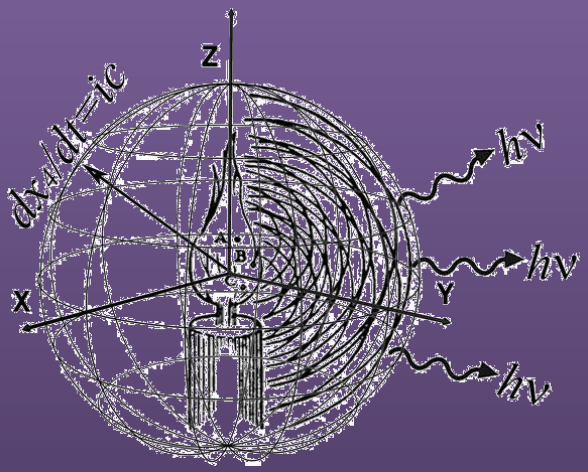
Quantum probabilistic nature

Classical world is Deterministic:
Knowing the position and velocity of all objects at a particular time
Future can be predicted using known laws of force and Newton's laws of motion.

Quantum World is Probabilistic:
Impossible to know position and velocity with certainty at a given time.
Only probability of future state can be predicted using known laws of force and equations of quantum mechanics.

Quantum mechanics does not predict the position of an electron exactly, rather, it provides only a ***probability*** as to where the electron will be found. We shall illustrate the probability aspect in terms of the system of an electron confined to motion along a line of length *L*. ***Quantum mechanical probabilities are expressed in terms of a distribution function***.

Each particle may be described by a wave function $\Psi(x,y,z,t)$, real or complex, associated to a probability, having a single value when position (x,y,z) and time (t) are defined. If it is not time-dependent, it is called stationary. Generally if the function Ψ is associated to an exponential form it represents a flow of particles: a plane wave.



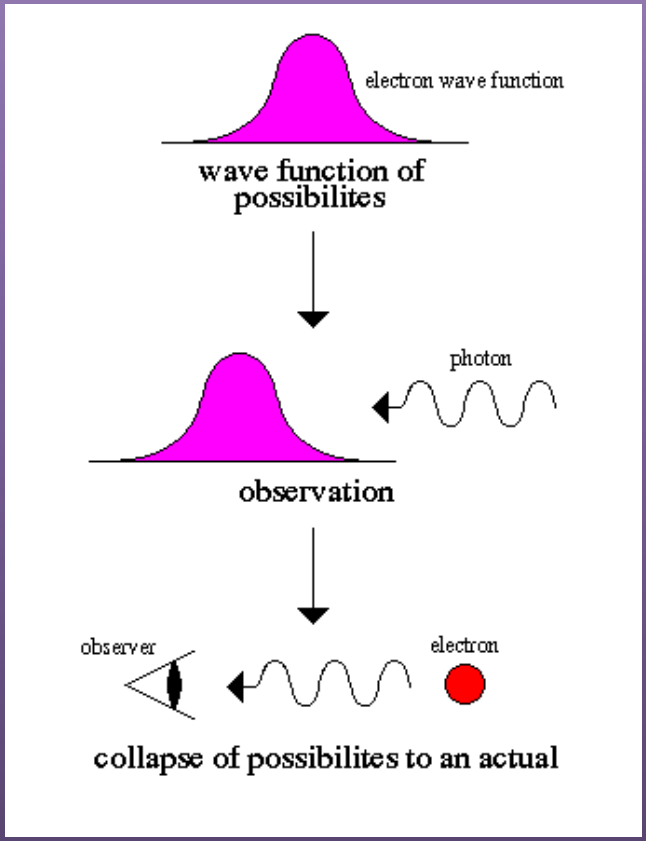


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Wave functions

The quantum wave function Ψ is a mathematical description of the superpositions associated with a quantum entity at any particular moment. The wave function, also called Schrodinger's Equation, is a mathematical description of all the possibilities for an object.

The key point to the wave function is that the position of a particle is only expressed as a probability *until* a measurement is made. For example, striking an electron with a photon results in a position measurement and we say that the wave function has `collapsed' (the wave nature of the electron converted to a particle nature). Its form is complex using partial derivatives.





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Schrödinger equation

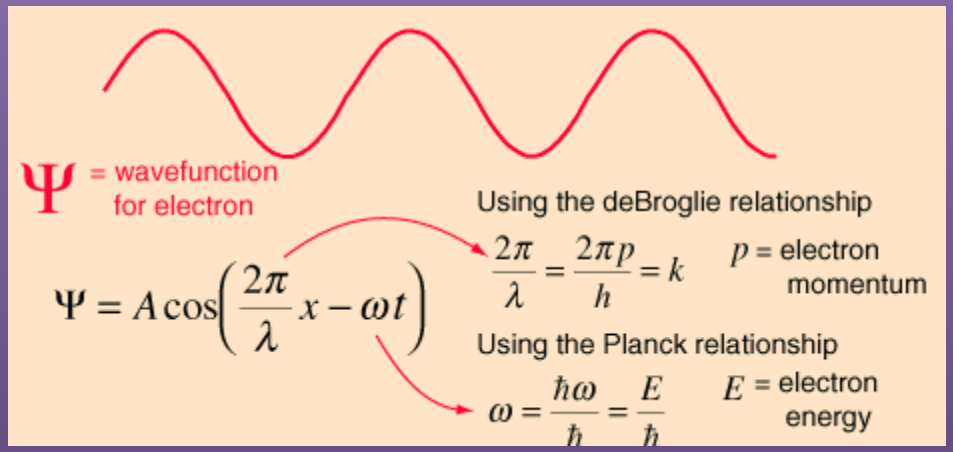
In **1925 Erwin Schrödinger** formulated a wave equation. The Schrödinger equation is used to find the allowed energy levels of quantum mechanical systems (such as atoms). The associated wavefunction Ψ gives the probability of finding the particle at a certain position. It is the time dependent equation used for describing progressive waves, applicable to the motion of free particles and the time independent form of this equation used for describing standing waves, within the limits of negligible relativistic effects.

In QM, a particle is not localized but has a probability to be in a given volume, in certain time that can be calculated by

$$\Delta p = \Psi^2 \Delta V \Delta t$$

that is the probability of finding the particle in the volume ΔV **in a time** Δt.

Around one point in space, $|\Psi(x,y,z,t)|^2$ is called density of probability and it is the ratio: $\Delta p/\Delta V\Delta t = \Psi^2 \rightarrow \Psi$ has the dimension of $L^{-1/3}$



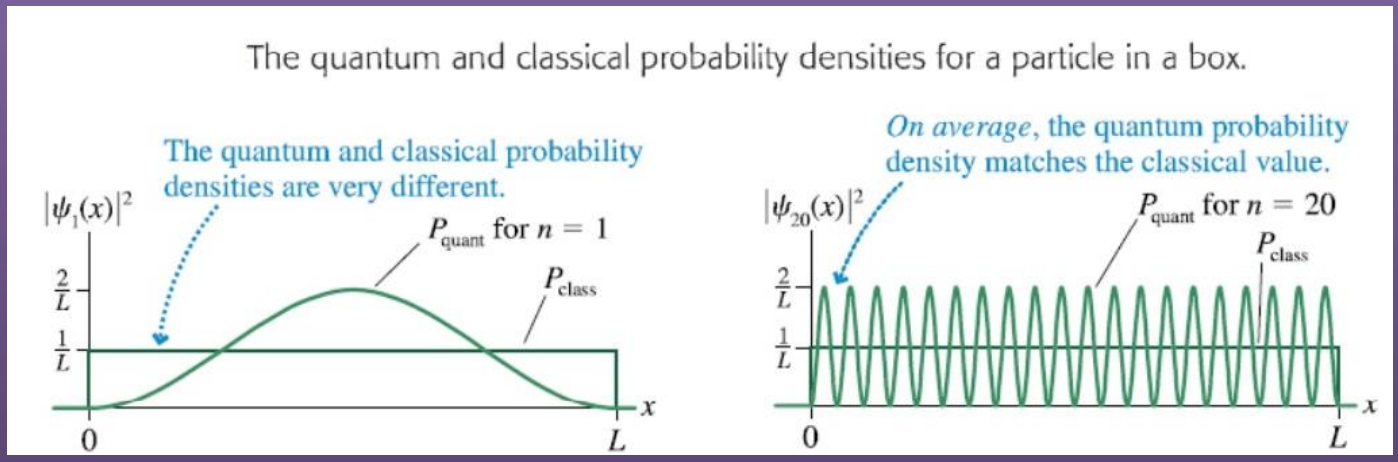
Its form is complex using partial derivatives.



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Correspondence principle

Formulated in **1923** by the Danish physicist **Niels Bohr**, the principle states that: ***the behavior of systems described by the theory of quantum echanics reproduces classical physics in the limit of large quantum numbers.*** In other words, it says that for large orbits and for large energies, quantum calculations must agree with classical calculations. It is also used more generally, to represent the idea that a new theory should reproduce the results of older well-established theories (which become limiting cases) in those domains where the old theories work.
Classical quantities appear in quantum mechanics in the form of expected values of observables.





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Learning Check 4

A. What does the complementary principle mean?

1) cannot observe both the wave and particle aspects

2) light interference in diffraction

B. Density of probability represents:

1) the product $\Delta V \Delta t$

2) the ratio $\Delta p / \Delta V \Delta t$

C. According to the quantum tunneling:

1) A particle can overcome potential barrier

2) a wave can enter in a tunnel



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Solution 4

A. What does the complementary principle mean?

1) cannot observe both the wave and particle aspects

B. Density of probability represents:

2) the ratio $\Delta p/\Delta V\Delta t$

C. According to the quantum tunneling:

1) a particle can overcome potential barrier



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